1. (20 points) Prove that $\sqrt{13}$ is irrational.

Note: There is a theorem which says “The positive integer $k$ has no rational square root if $k$ is not the square of an integer.” If you choose to use this theorem in your proof, you must give a proof of the theorem first.
2. (20 points) Use the Inclusion-Exclusion Principle to determine how many permutations of the set \{1,2,3,4,5\} have no even number as a fixed point.
3. (a) (10 points) Suppose that $x$ is irrational, $a$ is rational and $a \neq 0$. Prove that $ax - a^2$ must be irrational.

(b) (10 points) Give an example to show that the statement “If $x$ is irrational and $y$ is irrational, then $xy$ must be irrational.” is false.
4. (a) (10 points) State the definition of “The sequence \(< a_n >\) of real numbers converges to the limit \(L \in \mathbb{R}\).”

(b) (10 points) Use the definition from part (a) to prove that the sequence \(< \frac{n}{n+1} >\) converges to 1.
5. (20 points) Suppose $< a_n >$ is a sequence of real numbers which converges to $A \in \mathbb{R}$ and $< b_n >$ is a sequence of real numbers which converges to $B \in \mathbb{R}$. Prove that the sequence $< a_n + b_n >$ converges to $A + B$. 