11.7/2 \[3x + 2y = 30 \quad (z = \text{anything})\]

This is a plane with normal vector \((3, 2, 0)\).

\[3(x-10) + 2(y-0) + 0(z-0) = 0\]

It goes through the point \((10, 0, 0)\).
The plane can also be thought of as a cylinder on the line \(3x + 2y = 30\) in the \(xy\)-plane.

11.7/4 \[y^2 = x^2 - 9. \quad \text{This is a cylinder on the curve } x^2 - y^2 = 9, \text{ which is a hyperbola.}\]
11.7/6 \hspace{1cm} z = 4x^2 + 4y^2

\[ x^2 + y^2 = \frac{z}{4} \] \hspace{1cm} This is an elliptic paraboloid.

For \( z = z_0 \) constant, the trace is a circle.
For \( x = x_0 \) constant or for \( y = y_0 \) constant, the trace is a parabola.

11.7/24 \hspace{1cm} x^2 + 4y^2 + 2z^2 = 4

\[ \frac{x^2}{y^2} + \frac{y^2}{1} + \frac{z^2}{2} = 1 \] \hspace{1cm} ellipsoid
When $x = 0$, parabola opening down. When $y = 0$, parabola opening up. When $z = \text{constant}$, hyperbola.

This is a hyperboloid.
11.7/26 \( \frac{x^2}{9} - \frac{y^2}{9} - \frac{z^2}{1} = 1 \) \( \text{hyperboloid of two sheets} \)

11.7/28 \( y^2 + 4x^2 - 9z^2 = 36 \) \( \frac{y^2}{9} + \frac{x^2}{4} - \frac{z^2}{1} = 1 \) \( \text{hyperboloid of one sheet} \)
11.7/32.  $4x^2 + 9y^2 = 36$ is an ellipse in the $xy$ plane.

Rotate around $y$-axis.

We replace $x$ by $\sqrt{x^2 + z^2}$. So $x^2$ is replaced by $x^2 + z^2$.

$4x^2 + 4z^2 + 9y^2 = 36$

11.7/40 $z = 2x$ around $x$-axis.

Replace $z$ with $\sqrt{y^2 + z^2}$.

$\sqrt{y^2 + z^2} = 2x$ \quad $y^2 + z^2 = 2x^2$ \quad $z = 2x$ This is a double cone.
11.8/2 \quad r=3, \quad \theta = \frac{3\pi}{2}, \quad z = -1
so \quad x = 3 \cos \frac{3\pi}{2} = 3 \cdot 0 = 0
y = 3 \sin \frac{3\pi}{2} = 3 \cdot (-1) = -3
(x, y, z) = (0, -3, -1)

11.8/3 \quad r=2 \quad \theta = \frac{3\pi}{4}, \quad z = 3
x = 2 \cos \frac{3\pi}{4} = 2 \cdot \frac{-\sqrt{2}}{2} = -\sqrt{2}
y = 2 \sin \frac{3\pi}{4} = 2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2}
(x, y, z) = (-\sqrt{2}, \sqrt{2}, 3)

11.8/10 \quad \rho=4 \quad \phi = \frac{\pi}{6} \quad \theta = \frac{2\pi}{3}
x = 4 \sin \frac{\pi}{6} \cos \frac{2\pi}{3} = 4 \cdot \frac{1}{2} \cdot \frac{-1}{2} = -1
y = 4 \sin \frac{\pi}{6} \sin \frac{2\pi}{3} = 4 \cdot \frac{1}{2} \cdot \frac{\sqrt{3}}{2} = \sqrt{3}
z = 4 \cos \frac{\pi}{6} = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}
(x, y, z) = (-1, \sqrt{3}, 2\sqrt{3})
11.8/23 \( r = 5 \) cylindrical coordinates. This is a cylinder of radius 5.

11.8/24 \( \Theta = \frac{3\pi}{4} \) cylindrical or spherical coordinates

This is a plane containing the \( z \)-axis. It makes an angle of \( \frac{3\pi}{4} \) radians with the \( xz \) plane.

11.8/26 \( \rho = 5 \) spherical coords.

This is a sphere of radius 5, centered at the origin.

11.8/28 \( \phi = \frac{5\pi}{6} \) spherical coords.

This is a cone

\[
\begin{align*}
x &= \rho \cdot \frac{1}{2} \cos \Theta \\
y &= \rho \cdot \frac{1}{2} \sin \Theta \\
z &= \rho \cdot \frac{1}{\sqrt{3}} \\
z^2 &= 3x^2 + 3y^2
\end{align*}
\]
cylindrical coords.

11.8/33 \( r = 4 \cos \theta \). Multiply both sides by \( r \):

\[ r^2 = 4r \cos \theta \]
\[ x^2 + y^2 = 4x \]

\[ x^2 - 4x + 4 + y^2 = 0 + 4 \]
\[ (x-2)^2 + y^2 = 4 \] (\( z \) can be anything)

This is a cylinder of radius 2, with axis being the \( z \) vertical line through \( (x, y, z) = (2, 0, 0) \)

11.8/32 \( z^2 - 2r^2 = 4 \) cylindrical coords.
\[ z^2 - 2x^2 - 2y^2 = 4 \] This is a hyperboloid of two sheets.
11.8/40 \[ x^2 + y^2 = 2x \]

Cylindrical: \[ r^2 = 2r \cos \theta \]

Spherical: \[ \rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta = 2\rho \sin \phi \cos \theta \]

Since \( \cos^2 \theta + \sin^2 \theta = 1 \), this becomes

\[ \rho^2 \sin^2 \phi = 2\rho \sin \phi \cos \theta \]

\[ \rho \sin \phi = 2 \cos \theta \]

11.8/39 \[ x^2 + y^2 + z^2 = 25 \]

Cylindrical: \[ r^2 + z^2 = 25 \]

Spherical: \[ \rho^2 = 25 \] or \[ \rho = 5 \]

11.8/54 \[ y^2 - z^2 = 1 \], rotated around z-axis

We must replace \( y \) with \( \sqrt{x^2 + y^2} \) (See 11.7), which is \( r \) in cylindrical coords

\[ r^2 - z^2 = 1 \]