1. (3 points each part) Answer the questions by looking at the graph given of \( y = f(x) \). Note that the answer may be a number or \( \infty \) or \(-\infty\) or “does not exist”. Give an approximation if an exact answer is not possible. It is not necessary to show your work for this problem.

(a) Find \( f'(6) \). 
\[ = 2 \text{ approximately} \]

(b) Find \( f'(6) \). 
\[ \text{DNE} \]

(c) Find 
\[ \lim_{{x \to -8}} f(x) = -6 \]

(d) Find 
\[ \lim_{{x \to 6}} f(x) \text{ DNE} \]

(e) Find 
\[ \lim_{{x \to -2^+}} f(x) = \infty \]
2. (5 points each part) Evaluate each limit as a number, as $\infty$ or $-\infty$, or say “does not exist.” Show your work or give a brief explanation in words as to how you arrived at your answer. These limits should be done without the use of L'Hopital's Rule, which we have not covered yet.

(a)  
\[
\lim_{x \to -4} \frac{x + 1}{(x + 4)^2} = -\infty
\]
As $x \to -4$, $x + 1 \to -3$

\[
\frac{(x+4)^2}{x+4} \to 0 \quad \text{and is always positive.}
\]
So \(\frac{x+1}{(x+4)^2} \to -\infty\)

(b)  
\[
\lim_{x \to 1^-} \frac{x^2 - 1}{-(x-1)} = \lim_{x \to 1^-} \frac{(x-1)(x+1)}{-(x-1)}
\]
\[
= \lim_{x \to 1^-} \frac{x+1}{-1} = -2
\]

(c)  
\[
\lim_{x \to \infty} e^x = \infty
\]

\[
\text{Graph of } y = e^x
\]

(d)  
\[
\lim_{h \to 0} \frac{3 - (3+h)}{h} = \lim_{h \to 0} \frac{-h}{3(3+h)} = \lim_{h \to 0} \frac{-1}{3(3+h)} = -\frac{1}{9}
\]

\[
\text{Graph of } y = (3+h)
\]
3. (5 points each part) Find each derivative $f'(x)$. You may use any of the derivative rules and methods we have studied so far. (In particular, chain rule is allowed, although all of these problems may also be done without the chain rule.)

(a) $f(x) = 3x^2 - \sqrt{x} + e^x$. Find $f'(x)$.

$$f'(x) = 6x - \frac{1}{2}x^{-\frac{1}{2}} + e^x$$

(b) If $f(x) = xg(x)$ and $g(1) = 4$, $g'(1) = 7$, find $f'(1)$.

$$f'(x) = 1 \cdot g(x) + x \cdot g'(x)$$

$$f'(1) = g(1) + 1 \cdot g'(1)$$

$$= 4 + 7$$

$$= 11$$

(c) Find $f'(x)$ for

$$f(x) = \frac{2 \sin x}{3 - \tan x}.$$ 

$$f'(x) = \frac{2 \cos x (3 - \tan x) - 2 \sin x (-\sec^2 x)}{(3 - \tan x)^2}$$
4. (a) (5 points) State the definition of the derivative $f'(a)$.

\[ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \]

OR

\[ \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} \]

(b) (8 points) Use the definition of the derivative to show that $f'(1) = \frac{1}{2}$ for $f(x) = \sqrt{x}$.

\[ f'(1) = \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \]

\[ = \lim_{x \to 1} \frac{\sqrt{x} - 1}{x - 1} \cdot \frac{(\sqrt{x} + 1)}{(\sqrt{x} + 1)} \]

\[ = \lim_{x \to 1} \frac{x - 1}{(x - 1)(\sqrt{x} + 1)} \]

\[ = \lim_{x \to 1} \frac{1}{\sqrt{x} + 1} = \frac{1}{2} \]
5. (a) (5 points) State the Intermediate Value Theorem. Be sure to include hypotheses and conclusion ("if" and "then").

If \( f \) is continuous on \([a, b]\) and \( N \) is between \( f(a) \) and \( f(b) \),
then there is a \( c \in [a, b] \) with \( f(c) = N \).

(b) (7 points) Let \( f(x) = x^3 - 3x + 1 \). Use the Intermediate Value Theorem to show that there is a number \( c \) between 1 and 2 such that \( f(c) = 0 \). You should write in sentences (in addition to mathematical expressions) and be sure to verify, in writing, the hypotheses of the theorem.

\( f \) is continuous because it is a polynomial.

\[ f(1) = 1 - 3 + 1 = -1 \]
\[ f(2) = 8 - 6 + 1 = 3 \]

0 is between \( f(1) \) and \( f(2) \) so by IVT, there is a \( c \in (1, 2) \) with \( f(c) = 0 \).
6. (10 points) Find \( \frac{dy}{dx} \) for \( y = 2\cos x \). Then find the equation of the tangent line to the graph of \( y = 2\cos x \) at the point \((0, 2)\).

\[
\frac{dy}{dx} = -2\sin x
\]

At \( x = 0 \), \( \frac{dy}{dx} = 0 \)

Eq \( y - 2 = 0 \) \((x - 0)\)

\[ y = 2 \]
7. (3 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If \( \lim_{x \to 1} \frac{f(x)-f(1)}{x-1} = 3 \), then \( \lim_{x \to 1} f(x) = f(1) \).

\( T \quad (\text{differentiable } \Rightarrow \text{ cont.}) \)

(b) If \( f(x) \leq g(x) \leq h(x) \) for all \( x \) and if \( \lim_{x \to 2} f(x) = 3 \) and if \( \lim_{x \to 2} h(x) = 4 \), then \( \lim_{x \to 2} g(x) \) exists and \( 3 \leq \lim_{x \to 2} g(x) \leq 4 \).

\( F \quad (\lim_{x \to 2} g(x) \text{ need not exist}) \)

(c) Polynomials are both continuous and differentiable for all \( x \).

\( T \)

(d) If \( f(x) \) and \( g(x) \) are differentiable, then

\[
\frac{d}{dx}[f(x)g(x)] = f'(x)g(x).
\]

\( F \quad (\text{use product rule instead}) \)

(e) If \( f(x) \) and \( g(x) \) are differentiable and if \( c \) is a constant, then

\[
\frac{d}{dx}[cf(x) - g(x)] = cf'(x) - g'(x).
\]

\( T \)
1. (3 points each part) Answer the questions by looking at the graph given of \( y = f(x) \). Note that the answer may be a number or \( \infty \) or \( -\infty \) or "does not exist". Give an approximation if an exact answer is not possible. It is not necessary to show your work for this problem.

(a) Find
\[
\lim_{x \to -6} f(x) = -6
\]

(b) Find
\[
\lim_{x \to -2^+} f(x) = \infty
\]

(c) Find
\[
\lim_{x \to 6} f(x) \quad \text{DNE}
\]

(d) Find \( f'(10) \): \( \frac{-5}{2} \) (approximately)

(e) Find \( f'(-2) \): \( \text{DNE} \)
2. (5 points each part) Evaluate each limit as a number, as $\infty$ or $-\infty$, or say “does not exist.” Show your work or give a brief explanation in words as to how you arrived at your answer. These limits should be done without the use of L’Hôpital’s Rule, which we have not covered yet.

(a) \[
\lim_{x \to -2} \frac{x + 3}{(x+2)^2} = \infty
\]
As \( x \to -2 \), \( x + 3 \to 1 \)
\((x+2)^2 \to 0 \) and is always positive.
So \( \frac{x+3}{(x+2)^2} \to \infty \)

(b) \[
\lim_{x \to -1} \frac{x^2 + 3x + 2}{|x+1|} = \lim_{x \to -1} \frac{(x+1)(x+2)}{-(x+1)} = \lim_{x \to -1} \frac{x+2}{-1} = -1
\]

(c) \[
\lim_{x \to \infty} e^{-x} = 0
\]

(d) \[
\lim_{x \to 4} \frac{x - 4}{(x - 4)(\sqrt{x} + 2)} = \lim_{x \to 4} \frac{1}{\sqrt{x} + 2} = \frac{1}{4 + 2} = \frac{1}{4}
\]
3. (5 points each part) Find each derivative $f'(x)$. You may use any of the derivative rules and methods we have studied so far. (In particular, chain rule is allowed, although all of these problems may also be done without the chain rule.)

(a) $f(x) = \sqrt[3]{x} + x^5 - 2e^x$. Find $f'(x)$.

$$f'(x) = x^{\frac{1}{3}} + 5x^4 - 2e^x$$

(b) If $f(x) = x^2g(x)$ and $g(2) = 3$, $g'(2) = -1$, find $f''(2)$.

$$f'(x) = 2x \cdot g(x) + x^2 \cdot g'(x)$$

$$f'(2) = 4 \cdot g(2) + 4 \cdot g'(2)$$

$$= 4 \cdot 3 + 4 \cdot (-1)$$

$$= 8$$

(c) Find $f'(x)$ for

$$f(x) = \frac{4 \tan x}{5 + \cos x}$$

$$f'(x) = \frac{4 \sec^2 x \left(5 + \cos x\right) - 4 \tan x \left(-\sin x\right)}{(5 + \cos x)^2}$$
4. (a) (5 points) State the definition of the derivative $f'(a)$.

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

OR

$$\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

(b) (8 points) Use the definition of the derivative to show that $f'(2) = \frac{-1}{4}$ for $f(x) = \frac{1}{x}$.

$$f'(2) = \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} = \lim_{x \to 2} \frac{\frac{2 - x}{2x}}{x - 2}$$

$$= \lim_{x \to 2} \frac{-1}{2x} = \frac{-1}{4}$$
5. (a) (5 points) State the Intermediate Value Theorem. Be sure to include hypotheses and conclusion ("if" and "then").

If \( f \) is continuous on \([a, b]\) and \( N \) is between \( f(a) \) and \( f(b) \),

then there is a \( c \in [a, b] \) such that \( f(c) = N \)

(b) (7 points) Let \( f(x) = x^4 - 2x^3 - 1 \). Use the Intermediate Value Theorem to show that there is a number \( c \) between \(-1\) and \( 1 \) such that \( f(c) = 0 \). You should write in sentences (in addition to mathematical expressions) and be sure to verify, in writing, the hypotheses of the theorem.

\( f \) is continuous on \([a, b] = [-1, 1]\) because \( f \) is a polynomial.

\[
\begin{align*}
    f(-1) &= (-1)^4 - 2(-1)^3 - 1 = 1 + 2 - 1 = 2 \\
    f(1) &= 1 - 2 - 1 = -2 \\
\end{align*}
\]

\( 0 \) is between \( f(-1) \) and \( f(1) \).

By IVT, there exists \( c \in (-1, 1) \) with \( f(c) = 0 \).
6. (10 points) Find $\frac{dy}{dx}$ for $y = 1 + \sin x$. Then find the equation of the tangent line to the graph of $y = 1 + \sin x$ at the point (0,1).

$$\frac{dy}{dx} = \cos x$$

At $x = 0$, $\frac{dy}{dx} = 1$.

$$(y - 1) = 1 (x - 0)$$

$$y = x + 1$$
7. (3 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If \( \lim_{x \to 1} f(x) = f(1) \), then \( \lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} \) exists.

\[ F \quad (f(x) = |x-1| \text{ is an example}) \]

(b) The functions \( f(x) = \sin x \) and \( g(x) = \cos x \) are both continuous and differentiable for all \( x \).

\[ T \]

(c) If \( f(x) \leq g(x) \leq h(x) \) for all \( x \) and if \( \lim_{x \to 2} f(x) = 3 \) and if \( \lim_{x \to 2} h(x) = 4 \), then \( \lim_{x \to 2} g(x) \) exists and \( 3 \leq \lim_{x \to 2} g(x) \leq 4 \).

\[ F \quad (\text{\( \lim_{x \to 2} g(x) \) might not exist}) \]

(d) If \( f \) and \( g \) are differentiable, then

\[
\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x).
\]

\[ T \]

(e) If \( f \) and \( g \) are differentiable, then

\[
\frac{d}{dx}\left[ \frac{f(x)}{g(x)} \right] = \frac{f'(x)}{g'(x)}.
\]

\[ F \quad (\text{use quotient rule instead}) \]