Ways to show that a critical point is an absolute max or absolute min.

Use these for the optimization problems in Section 4.7.

For all of these methods, we must have \( f(x) \) continuous and differentiable on an interval from \( a \) to \( b \). The interval can be open, closed, half-open, or infinite – any interval. Also, the point \( x=c \) must be the ONLY critical point for \( f \) in the interval (Method 3 is an exception).

**Method 1:** If \( f'(x)>0 \) for all \( a<x<c \) and if \( f'(x)<0 \) for all \( c<x<b \), then \( f \) has an absolute maximum at \( x=c \). If \( f'(x)<0 \) for all \( a<x<c \) and if \( f'(x)>0 \) for all \( c<x<b \), then \( f \) has an absolute minimum at \( x=c \).

**Method 2:** If \( f''(c)<0 \), then \( f \) has an absolute maximum at \( x=c \). If \( f''(c)>0 \), then \( f \) has an absolute minimum at \( x=c \).

**Method 3:** If the interval is a closed interval, you can check the endpoints and the critical point to see which gives the absolute maximum and which gives the absolute minimum for \( f \). (Same as you did in Section 4.1).

**Method 4:** If \( \lim_{x\to a^+} f(x) = \infty \) and if \( \lim_{x\to b^-} f(x) = \infty \), then \( f \) has an absolute minimum at \( x=c \). If \( \lim_{x\to a^-} f(x) = -\infty \) and if \( \lim_{x\to b^+} f(x) = -\infty \), then \( f \) has an absolute maximum at \( x=c \).

Remember these only work if \( x=c \) is the ONLY critical point in the interval!