You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.
1. (6 points each part) For each part, find $f'(x)$.

(a) 

$$f(x) = \frac{1}{(x^3 + 3)^3}$$

(b) $f(x) = \ln(x^2)$

(c) $f(x) = \tan^{-1} x$. 


2. (6 points) Find the function $f$ such that $f'(x) = 6x + 5$ and $f(1) = 9$.

3. (6 points) Use implicit differentiation to find $dy/dx$.

$$x^2y^2 + x \sin y = 4$$
4. (5 points each part) Find each limit. You may use any method. Show your work or briefly explain your reasoning - no credit for the answer alone.

(a) \[ \lim_{x \to 0} \frac{x - \sin x}{x^2 + \cos x - 1} \]

(b) \[ \lim_{x \to 0^+} \frac{\ln x}{x} \]
5. (5 points each part)

(a) Give the definition of “$f$ has a local maximum at $c$.”

(b) Give the definition of “$f$ has an absolute minimum at $c$.”
6. (15 points) If a total of 1200 square cm of cardboard is to be used to make a box with a square base and an open top, find the largest possible volume of the box.

*Important note:* for full credit, you must explain how you know that the volume you have found is the largest.
7. (5 points each part) In this problem, be sure to notice that the graph is the derivative $y = f'(x)$ and the questions are about the function $f(x)$. Use the given graph of $y = f'(x)$ to answer the questions.

(a) On what intervals is $f$ increasing and on what intervals is $f$ decreasing?

(b) At what values of $x$ does $f$ have a local maximum or local minimum?

(c) On what intervals is $f$ concave up and on what intervals is $f$ concave down?

(d) What are the $x$-values for the inflection points for $f$?
8. (3 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) The linear approximation to \( f(x) = x^5 \) at \( x = 1 \) is \( L(x) = 1 + 5(x - 1) \)

(b) There does not exist a function which is continuous and differentiable for all \( x \) and which has the properties \( f(1) = -2, f(3) = 0, \) and \( f'(x) > 1 \) for all \( x \).

(c) Suppose \( f \) is a function which is differentiable for all \( x \) and which has a root at \( c \). If we choose any initial approximation \( x_1 \) and apply Newton’s method, then the successive approximations \( x_1, x_2, x_3, x_4, \ldots \) will get closer and closer to the root \( c \).

(d) The Extreme Value Theorem says that if \( f \) is continuous on a closed interval, then \( f \) attains an absolute maximum value and an absolute minimum value on that closed interval.

(e) The critical points of \( f \) are exactly the points where \( f'(x) = 0 \).