DO NOT OPEN EXAM UNTIL TOLD TO DO SO.
SIT IN THE SEAT CIRCLED BELOW.

You may not use any books or notes or calculator. There are 100 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written.
1. (6 points each part) For each part, find $f'(x)$.

(a) $f(x) = \sqrt[4]{1 + 2x + x^3}$

(b) $f(x) = x \ln x$

(c) $f(x) = \sin^{-1} x$. 
2. (6 points) Find the function $f$ such that $f'(x) = 2 - 4x$ and $f(1) = 7$.

3. (6 points) Use implicit differentiation to find $dy/dx$.

$$x^4 \sin y + x^3 y^3 = 6$$
4. (5 points each part) Find each limit. You may use any method. Show your work or briefly explain your reasoning - no credit for the answer alone.

(a) 
\[
\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}
\]

(b) 
\[
\lim_{x \to 0^+} \frac{x}{\ln x}
\]
5. (5 points each part)

(a) Give the definition of “f has an absolute maximum at c.”

(b) Give the definition of “f has a local minimum at c.”
6. (15 points) A farmer wants to fence an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. how can he do this so as to minimize the cost of the fence?

Important note: for full credit, you must explain how you know that your answer gives the minimum cost.
7. (5 points each part) In this problem, be sure to notice that the graph is the derivative $y = f'(x)$ and the questions are about the function $f(x)$. Use the given graph of $y = f'(x)$ to answer the questions.

(a) On what intervals is $f$ increasing and on what intervals is $f$ decreasing?

(b) At what values of $x$ does $f$ have a local maximum or local minimum?

(c) On what intervals is $f$ concave up and on what intervals is $f$ concave down?

(d) What are the $x$-values for the inflection points for $f$?
8. (3 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) There exists a function which is continuous and differentiable for all $x$, with the properties $f(1) = -2$, $f(3) = 0$, and $f'(x) > 2$ for all $x$.

(b) The linear approximation to $f(x) = x^5$ at $x = 1$ is $L(x) = 1 + 5(x - 1)$.

(c) The Extreme Value Theorem says that if $f$ is continuous on a closed interval, then $f$ attains an absolute maximum value and an absolute minimum value on that closed interval.

(d) Suppose $f$ is a function which is differentiable for all $x$ and which has a root at $c$. If we choose any initial approximation $x_1$ and apply Newton’s method, then the successive approximations $x_1, x_2, x_3, x_4, \ldots$ will get closer and closer to the root $c$.

(e) The critical points of $f$ are exactly the points where $f'(x) = 0$. 