1. (3 points each part) Answer the questions by looking at the graph given of \( y = f(x) \).

(a) List all of the numbers at which \( f \) is not continuous.
\[
\chi = 1, 4
\]

(b) List all of the numbers at which \( f \) is not differentiable.
\[
\chi = -2, 1, 4
\]

(c) Find \( \lim_{x \to -1} f(x) \)
\[
1
\]

(d) Find \( \lim_{x \to \infty} f(x) \)
\[
0 \text{ (or about 0.5 is also counted correct)}
\]

(e) Find \( \lim_{x \to -4} f(x) \)
\[
\infty
\]
2. (5 points each part) Evaluate each limit as a number, as \( \infty \) or \(-\infty\), or say “does not exist.” Show your work or give a brief explanation in words as to how you arrived at your answer. These limits should be done without the use of L'Hopital's Rule, which we have not covered yet.

(a) 
\[
\lim_{x \to 2^-} \ln(x^2 - 4)
\]
When \( x \) is slightly larger than 2, \( x^2 - 4 \) is slightly larger than 0 and \( x^2 - 4 \to 0 \) as \( x \to 2^+ \).

So \( \lim_{x \to 2^+} \ln(x^2 - 4) = -\infty \)

(b) 
\[
\lim_{x \to -4} \frac{1 + \frac{1}{x}}{4 + x}
\]
\[
= \lim_{x \to -4} \frac{\frac{4 + x}{x}}{4x} = \lim_{x \to -4} \frac{1}{4x} = \frac{-1}{16}
\]

(c) 
\[
\lim_{x \to \infty} \frac{\frac{1}{x^5} - \frac{1}{x^3} + \frac{1}{x}}{\frac{1}{x^4} + \frac{1}{x^2} + 1}
\]
\[
= \lim_{x \to \infty} \frac{0}{1} = 0
\]

(d) 
\[
\lim_{x \to \infty} \cos x
\]
does not exist, because \( \cos x \) oscillates between -1 and 1 and does not approach any single number.
3. (5 points each part) Find each derivative $f'(x)$. You may use any of the derivative rules and methods we have studied so far.

(a) 
$f(x) = \frac{1}{x^{10}} + 4\pi^2$

$f'(x) = -10x^{-11}$. Remember $4\pi^2$ is a constant, so its derivative is $0$.

(b) 
$f(x) = x^3 \cos x$

Product rule.
$f'(x) = 3x^2 \cos x + x^3(-\sin x)$

(c) 
$f(x) = \frac{x}{e^x}$

Quotient rule.
$f'(x) = \frac{1 \cdot e^x - x e^x}{(e^x)^2}$

$= \frac{1-x}{e^x}$
4. (a) (3 points) State the definition of the derivative $f'(a)$.

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

or

$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

(b) (8 points) Use the definition of the derivative to show that $f'(1) = 6$ for $f(x) = 3x^2$.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{3(1+h)^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{3 + 6h + 3h^2 - 3}{h}$$

$$= \lim_{h \to 0} \frac{6h + 3h^2}{h}$$

$$= \lim_{h \to 0} 6 + 3h = 6.$$
5. (a) (5 points) State the Squeeze Theorem.

\[ \text{If } f(x) \leq g(x) \leq h(x) \quad \text{for all } x \text{ near } a, \ x \neq a \]

and \( \lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L \)

\[ \text{then } \lim_{x \to a} g(x) = L \]

(b) (7 points) Use the Squeeze Theorem to show that

\[ \lim_{x \to 0} \left( \sin^2 x \right) \left( \cos \frac{1}{x} \right) = 0. \]

You should write in sentences (in addition to mathematical expressions) and be sure to verify, in writing, the hypotheses of the theorem.

For all \( x \neq 0 \),

\[ -1 \leq \cos \frac{1}{x} \leq 1. \]

So \( -\sin^2 x \leq \sin^2 x \cos \frac{1}{x} \leq \sin^2 x \)

\[ \lim_{x \to 0} (-\sin^2 x) = \lim_{x \to 0} \sin^2 x = 0. \]

Therefore, by the Squeeze Theorem,

\[ \lim_{x \to 0} \left( \sin^2 x \right) \left( \cos \frac{1}{x} \right) = 0 \]
(10 points) Sketch the graph of a single function $f$ having all of the following properties. You do not have to give a formula for $f(x)$, just a graph.

- $f(0) = 0$
- $f'(0) = 0$
- $f'(-1) = -1$
- $f'(1) = 3$
- $f'(2) = -1$

Here is one such graph:
7. (3 points each part) For this problem, answer true or false for each part. You do not need to show work or give any reason, and there is no partial credit on this problem.

(a) If \( \lim_{x \to 3^+} f(x) = 4 \) and \( \lim_{x \to 3^-} f(x) = 5 \), then \( \lim_{x \to 3} f(x) \) does not exist.

True. \( (\lim_{x \to 3^+} f(x) \text{ exists only if } \lim_{x \to 3} f(x) = \lim_{x \to 3^-} f(x)) \)

(b) If \( f \) is discontinuous at \( x = 0 \) and if \( f(0) = 8 \), then the discontinuity must be removable.

False. (The discontinuity at \( x = 0 \) is removable only if \( \lim_{x \to 0} f(x) \text{ exists } \)).

(c) If \( f \) is differentiable at \( x = 1 \), then \( \lim_{x \to 1} f(x) = f(1) \).

True. (\( f \) differentiable at \( x = 1 \) \( \Rightarrow \) \( f \) continuous at \( x = 1 \) \( \Rightarrow \lim_{x \to 1} f(x) = f(1) \)).

(d) Crickets chirp more frequently when the temperature is higher. Suppose that \( f(x) \) is the average number of chirps per hour when the temperature is \( x \) degrees Fahrenheit and suppose that \( f(70) = 5 \). A correct interpretation is if the temperature rises from 70 degrees to 71 degrees, then one would expect about 5 more chirps per hour on average.

True. (rate of change - see Section 2.8)

(e) If \( f(x) \) is defined for all \( x \), then \( f \) is continuous at all \( x \).

False. (\( f \) is continuous at each point \( a \) only if \( \lim_{x \to a} f(x) = f(a) \)).