Time: 3 hours. You may not use any books or notes or calculator. There are 150 points possible. To get any credit, you must show your work. Unless indicated, you do not need to simplify your answers. Partial credit will be based only on what is actually written on the paper. All intermediate steps should be correct as written, including correct notation.

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The following formulas are provided for your use. You may tear this page out if you wish.

1. \[ \sum_{i=1}^{n} c = cn \]

2. \[ \sum_{i=1}^{n} i = \frac{n(n + 1)}{2} \]

3. \[ \sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6} \]
1. (4 points each part) Evaluate each of the following limits. Any (correct) method may be used.

(a) \[ \lim_{x \to 2} \frac{e^{x+1}}{x} \]

(b) \[ \lim_{x \to 1^+} \frac{x}{1 - x} \]

(c) \[ \lim_{x \to \infty} \frac{x^3}{e^x} \]

(d) \[ \lim_{x \to 0^+} (1 - 2x)^{1/x} \]
2. (5 points each part) Evaluate each derivative. Any (correct) method may be used.

(a) \( \frac{d}{dx}(x^2 \cos x) \)

(b) \( \frac{d}{dx} \sqrt[3]{\frac{x}{\ln x}} \)

(c) Find \( \frac{dy}{dx} \) for 
\[ 1 + x = e^{xy^2}. \]
3. (5 points each part) Evaluate each definite or indefinite integral.

(a) \[ \int \frac{1}{1 + x^2} \, dx \]

(b) \[ \int \frac{1}{5 - 3x} \, dx \]

(c) \[ \int_{1}^{2} \frac{e^{1/x}}{x^2} \, dx \]
4. (10 points) A particle is moved along the $x$-axis by a force that measures $3(1 + x)^{-2}$ pounds $x$ feet from the origin. Find the work done in moving the particle from the origin to a distance of 6 feet.

5. (10 points) Find the average value of $f(x) = 1 + x^2$ on the interval $[-2, 1]$. 
6. (10 points) Find the equation of the tangent line to the curve \( y = x^2 - x^3 \) at the point where \( x = 1 \).

7. (10 points) Find the absolute minimum and absolute maximum of \( f(x) = 3x^2 - 12x + 5 \) on \([1, 3]\).
8. (a) (5 points) Give the definition of continuity (what is meant by “$f$ is continuous at $a$?”).

(b) (5 points) Draw the graph of a function defined on $[0, 4]$ which has a removable discontinuity at $x = 1$, a jump discontinuity at $x = 2$ and which is continuous at all other points of $[0, 4]$. (You need not give a formula for $f(x)$ - just draw the graph.)
9. (a) (5 points) Give the definition of $f'(x)$.

(b) (5 points) Use the definition of $f'(x)$ to show that the derivative of $f(x) = \sqrt{x}$ is $f'(x) = \frac{1}{2}x^{-1/2}$.
10. (a) (5 points) State the Mean Value Theorem.

(b) (10 points) Use the Mean Value Theorem (or Rolle’s Theorem, if you prefer) to show that the function \( f(x) = 2x - 1 - \cos x \) cannot have 2 zeroes.
11. (10 points) The definition of definite integral is

\[ \int_{a}^{b} f(x) \, dx = \lim_{n \to \infty} \sum_{i=1}^{\infty} f(x_i^*) \Delta x. \]

(I have not included the meaning of all the notation - this is left to you to interpret as you do the problem.) Use this definition to show that

\[ \int_{0}^{4} (1 + 3x) \, dx = 28. \]

You must use the definition - no credit for other methods.
12. (10 points) A cylindrical tank with radius 4 m. is being filled with water at a rate of 4 cubic meters per minute. How fast is the height of the water increasing?

13. (3 points each part) Answer true (T) or false (F). No explanation is necessary

(a) If $f$ is continuous for all $x$, then $f$ has an antiderivative.

(b) If $\lim_{x \to 0} [f(x) + g(x)]$ exists, then $\lim_{x \to 0} f(x)$ and $\lim_{x \to 0} g(x)$ must both exist.

(c) If $f$ is continuous for all $x$, then $\int_a^b x f(x) \, dx = x \int_a^b f(x) \, dx$. 
End of test. Use this blank page for extra scratch paper if needed.