Midterm 1 review: Chapters 1-2.

- Find fixed points of 1D maps
- Analyze/classify stability with derivative test
- Show stability when derivative test fails
- Draw and/or analyze a cobweb plot
- Find period-2 points (or higher period), analyze/classify stability
- Know what the logistic map is
- Bifurcation diagram
- Periodic table
- Sensitive dependence on initial conditions
- Itineraries: write/draw them, interpret them, understand how the list of symbols works, how it changes between $x$ and $f(x)$ and $f^k(x)$...
- Transition graph
- Steps of Challenge 1
- Find fixed points of 2D or higher degree maps
- Find periodic points of 2D or higher degree maps
- Analyze/classify stability of fixed points or periodic points of 2D or higher degree maps
- Know what the Hénon Map is
- Manifolds - know what they are, stable and unstable manifolds of saddle points...
- Find manifolds for linear maps with saddle points
- Find manifold in neighborhood of saddle point of nonlinear map
- Prove something is a stable or unstable manifold
- Find eigenvalues and eigenvectors of matrix
- Homoclinic point: know it is a point of intersection of stable and unstable manifolds of a given saddle point that is not at a fixed point - if one exists then infinite must exist because a homoclinic point must map to another homoclinic point...

Things that hint towards chaos:

- Sensitive dependence on initial conditions
- Infinite number of periodic orbits
- Existence of homoclinic points
- Existence of period-3 orbits

Suggested practice problems (special emphasis on ones that were actually assigned (or similar) and ones we did in class (or similar):

- T1.3, T1.5-T1.9, T1.11, T1.13-T1.16
- Ch 1 Exercises 1.1-1.7, 1.9, 1.10, 1.12, 1.14
- T2.1, T2.2, T2.5-T2.7, T2.9
- Ch 2 Exercises 2.1-2.7

Additional problems from class Thurs: (I'll try to post solutions sometime Monday)

1. Given a 2D linear map $R(\vec{x}) = A\vec{x}$, $A = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix}$, $\alpha \neq 2\pi n$.
   (a) Find fixed points of $R$
   (b) Analyze stability. What is actually happening to solutions?
   (c) Let $\alpha = 2\pi/m$, $m \in \mathbb{Z}$. Find periodic points. What is their period?

2. Now try $X(x, y) = (-y\sqrt{x^2 + y^2}, x\sqrt{x^2 + y^2})$.
   (a) Show that $(0, 0)$ is the only fixed point.
   (b) Analyze the stability of $(0, 0)$.
   (c) Show that $(1, 0)$ is a periodic fixed point by finding the other points of its periodic orbit. What is the period?
   (d) Can you analyze its stability?
   (e) Show that actually $(\cos \alpha, \sin \alpha)$ is a periodic fixed point for all $\alpha$.

3. Given a 1D nonlinear map: $f(x) = r - x^2$.
   (a) Find fixed point(s)
   (b) For what values of $r$ are one or more fixed points stable?
   (c) Find period 2 points for $r = 3$.
   (d) Is the period-2 orbit stable when $r = 3$?

4. What is the area of the region mapped to by the unit square $0 < x, y < 1$ by the map $f(\vec{x}) = A\vec{x}$ where $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $a, b, c, d \in \mathbb{R}$?