Summary of the standard discrete densities

Bernoulli(p): X=result of a single Bernoulli trial.

$$f(k) = P(X = k) = \begin{cases} 1-p & \text{if } k = 0 \\ p & \text{if } k = 1 \\ 0 & \text{otherwise} \end{cases},$$

$E[X] = p, \quad \mathrm{Var}(X) = p(1-p).$

Binomial(n, p): X=number of successes in n independent Bernoulli trials.

$$f(k) = P(X = k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{if } k = 0, 1, 2, \ldots, n \\ 0 & \text{otherwise} \end{cases},$$

$E[X] = np, \quad \mathrm{Var}(X) = np(1-p).$

Note. Binomial$(1, p)$=Bernoulli(p).

Geometric(p): X=number of trials for the first success, in an infinite sequence of Bernoulli trials.

$$f(k) = P(X = k) = \begin{cases} p(1-p)^{k-1} & \text{if } k = 1, 2, 3, \ldots \\ 0 & \text{otherwise} \end{cases},$$

$E[X] = \frac{1}{p}, \quad \mathrm{Var}(X) = \frac{1-p}{p^2}.$

Useful formula: $P(X \geq k) = (1-p)^{k-1}$

Negative Binomial(r, p): X=number of trials for achieving r successes.

$$f(k) = P(X = k) = \begin{cases} \binom{k-1}{r-1} p^r (1-p)^{k-r} & \text{if } k = r, r+1, r+2, \ldots \\ 0 & \text{otherwise} \end{cases},$$

$E[X] = \frac{r}{p}, \quad \mathrm{Var}(X) = r\frac{1-p}{p^2}.$

Note. Negative Binomial$(1, p)$=Geometric(p).

Poisson(λ): X=number of events occurring during a time interval of length 1, given that events occur at an average rate of λ per unit time.

$$f(k) = P(X = k) = \begin{cases} e^{-\lambda}\lambda^k/k! & \text{if } k = 0, 1, 2, \ldots \\ 0 & \text{otherwise} \end{cases},$$

$E[X] = \lambda, \quad \mathrm{Var}(X) = \lambda.$

Note that $X \sim$ Poisson(λT) is used for the number of events occurring during a time interval of length T, given that events occur at an average rate of λ per unit time.

Remark. $X \sim$ Poisson(λ) with $\lambda = np$ provides a good approximation to Binomial(n, p), assuming n is large and np is moderate.

Hypergeometric(n, N, m): X=number of white balls in a sample of n balls taken without replacement from an urn of N balls, of which m are white and $N - m$ are black.

$$f(k) = P(X = k) = \begin{cases} \binom{m}{k}\binom{N-m}{n-k} \Big/ \binom{N}{n} & \text{if } k = 0, 1, 2, \ldots, n, \\ 0 & \text{otherwise,} \end{cases}$$

$E[X] = n\frac{m}{N}, \quad \mathrm{Var}(X) = \frac{N-n}{N-1}\, n\frac{m}{N}\left(1 - \frac{m}{N}\right).$