Summary of the standard continuous densities

Uniform(a,b): X=randomly chosen point in the interval (a,b). Density

$$f(x)=\begin{cases}\frac{1}{b-a} & \text{if } a<x<b,\\ 0 & \text{otherwise,}\end{cases}$$

$E[X]=\frac{b+a}{2}$, $\quad \mathrm{Var}(X)=\frac{(b-a)^2}{12}$.

Normal(μ,σ^2): X=random fluctuation arising from many causes. Density

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-(x-\mu)^2/2\sigma^2}, \qquad -\infty<x<\infty,$$

$E[X]=\mu$, $\quad \mathrm{Var}(X)=\sigma^2$. When $\mu=0$ and $\sigma=1$, one obtains the standard normal density $\phi(x)=\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$.

Useful formula: $P(a\le X\le b)=\Phi(\frac{b-\mu}{\sigma})-\Phi(\frac{a-\mu}{\sigma})$ where Φ is the distribution function of the standard normal density: $\Phi(z)=\int_{-\infty}^{z}\phi(x)\,dx$.

Important result: the Central Limit Theorem (Section 8.3) says that the distribution of outcomes is approximately normal, after many independent repetitions of a random experiment, no matter what the experiment is!

Exponential(λ): X=waiting time until the first event, if there are λ events per unit time on average. Density

$$f(x)=\begin{cases}\lambda e^{-\lambda x} & \text{if } x\ge 0,\\ 0 & \text{otherwise,}\end{cases}$$

$E[X]=\frac{1}{\lambda}$, $\quad \mathrm{Var}(X)=\frac{1}{\lambda^2}$.

Useful formula: $P(X>x)=e^{-\lambda x}$ for $x\ge 0$.

(*Discrete analogue* of Exponential(λ) is Geometric(p) with $p=1-e^{-\lambda}$.)

Gamma(α,λ): X=waiting time until the α^{th} event (when α is a positive integer), if there are λ events per unit time on average. Density

$$f(x)=\begin{cases}\frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\Gamma(\alpha)} & \text{if } x\ge 0,\\ 0 & \text{otherwise,}\end{cases}$$

$E[X]=\frac{\alpha}{\lambda}$, $\quad \mathrm{Var}(X)=\frac{\alpha}{\lambda^2}$.

Note. Gamma$(1,\lambda)$=Exponential(λ).

Cauchy: X=tangent of a uniformly randomly chosen angle between $-\pi/2$ and $\pi/2$. Density

$$f(x)=\frac{1}{\pi(1+x^2)},$$

$E[X]=0$, $\quad \mathrm{Var}(X)=+\infty$.

Exercise. Sketch each of the density functions $f(x)$, to get a feel for which X-values are more likely to occur, in each case.