Graphical Analysis of Phase Transition in XY Model

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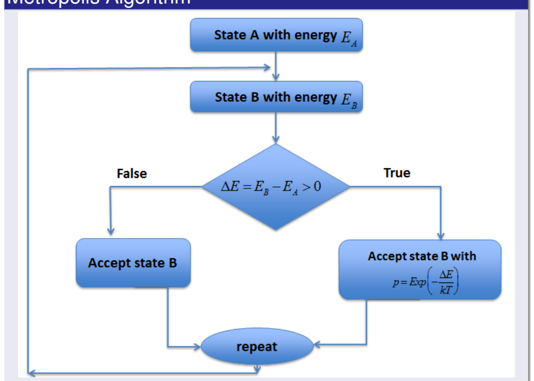
Mean Field Theory in XY Model

- *XY Model*: Every spin is on the unit circle
- Graphically, it is similar to complete graph, which means that every two distinct particles interact with each other
- *Mean Field Theory*: The study of average effect of all the particles on a particular particle
- The Hamiltonian is given by: $H(\sigma) = -\Sigma J_{ij} < \sigma_i, \sigma_j >= -\frac{1}{2N}\Sigma \cos(\theta_i - \theta_j)$

Ordered Parameters' Formulas

- Average Energy: $< E >= \frac{H(\sigma)}{N}$
- Magnetization: $M = \frac{\Sigma \sigma_i}{N}$
- Specific Heat: $C_v = \frac{1}{kT^2}[< E^2 > - < E >^2]$ scaled variance of Energy, where k is Boltzmann constant, T is Temperature
- Susceptibility: $\chi^2_M = \frac{1}{T}[< M^2 > - < M >^2]$

Metropolis Algorithm



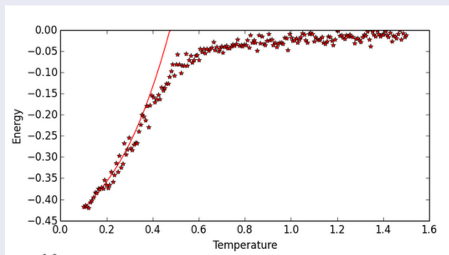
- The algorithm is an 'accept-reject' algorithm where we calculate the difference of the total energy in original configuration and in new configuration;
- If energy of new configuration is larger, we accept new configuration with probability $Exp(-\frac{\delta(E)r}{kT})$; if energy of new configuration is smaller, we accept new configuration with probability 1

Phase Transition

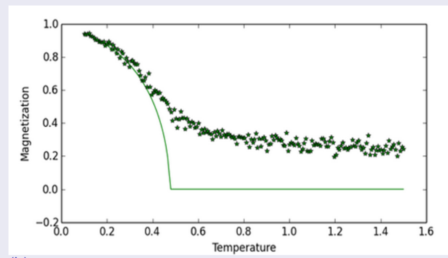
- Physically: Transformation of a thermodynamic system from one phase or state of matter to another one by heat transfer
- Mathematically: The study of non-analycity of free energy, which is given by

$$U \equiv < E >= \sum_r P_r E_r = -\frac{\partial \log Z}{\partial \beta}$$

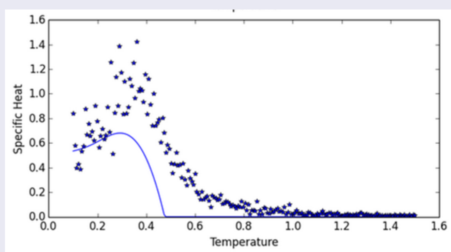
Ordered Parameters' Graphical Analysis



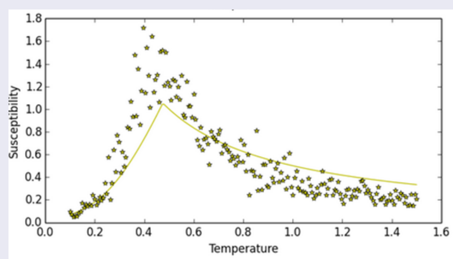
(a) As Temperature rises from 0 to 0.5, Average Energy increases exponentially. After critical temperature 0.5, the Average Energy becomes stable



(b) Before critical temperature 0.5, the increase in temperature leads to decrease in Magnetization; after critical temperature magnetization tends to zero, which indicates at high temperature, particles move in random directions



(c) Specific heat experienced a parabola change before critical temperature; after critical temperature Specific Heat tends to 0 as Temperature increases

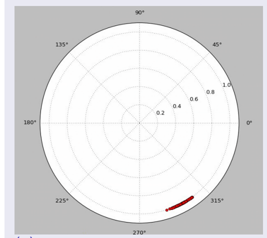


(d) Susceptibility increases as Temperature increases before critical Temperature; after it reaches a peak at critical temperature, Susceptibility decreases as Temperature decreases

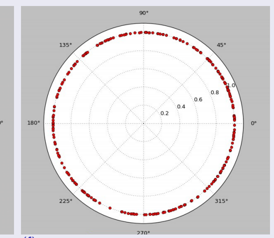
Average Spin Behavior in Different Temperature Regimes

- For low temperature, spins are all lined up in same directions,as T goes down, M increases, $|E|$ increases
- For high temperature, spins are pointed randomly in different directions, as T goes up, M decreases, $|E|$ decreases

Animated Simulation Plot



(e) $T = 0.01$; $E = -0.4945$; $M = 0.9983$; $\chi^2_M = 0.01$



(f) $T = 1.50$; $E = -0.0056$; $M = 0.0745$; $\chi^2_M = 0.37$

Applications of Superconductors

- Biomagnetism
- Magnetic-levitation
- SQUIDs (Superconducting Quantum Interference Device)

Future Work

- Study the correlations of spin behaviors under different temperature regions
- Numerical calculation for rate function and Large Deviation Principle for each regime
- Graphical analysis of limit theorems in subcritical/critical/supercritical regimes

Acknowledgements

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- Reference:
https://github.com/seanshahkarami/demos/blob/master/spindemo.py
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