

Third Homework Set — Solutions

Chapter 3

Problem 1 Let E be the event that at least one die lands on six, and let F be the event that the dice land of different numbers. Then $P(EF) = 2 \cdot \frac{1}{6} \cdot 56 = \frac{5}{18}$ and $P(F) = \frac{30}{36} = \frac{5}{6}$. Hence,

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{5}{18}}{\frac{5}{6}} = \frac{1}{3}.$$

Problem 5

$$\frac{6 \cdot 5 \cdot 9 \cdot 8}{15 \cdot 14 \cdot 13 \cdot 12} = \frac{6}{91}$$

Problem 6 Let A be the event that the sample drawn contains exactly three white balls. Let B be the event that the first and third ball drawn are white.

Without replacement

$$P(A) = 4 \frac{8 \cdot 7 \cdot 6 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9} \text{ and } P(AB) = 2 \frac{8 \cdot 7 \cdot 6 \cdot 4}{12 \cdot 11 \cdot 10 \cdot 9}, \text{ hence } P(B|A) = \frac{P(AB)}{P(A)} = \frac{1}{2}.$$

With replacement

$$P(A) = \binom{4}{3} \left(\frac{2}{3}\right)^3 \frac{1}{3} \text{ and } P(AB) = 2 \left(\frac{2}{3}\right)^3 \frac{1}{3}, \text{ hence } P(B|A) = \frac{P(AB)}{P(A)} = \frac{1}{2}.$$

Problem 9 Let E_i be the event that the ball drawn from the i -th urn is white, for $i = 1, 2, 3$. Let F be the event that exactly two white balls were drawn. Then

$$\begin{aligned} P(E_1|F) &= \frac{E_1 F}{P(F)} \\ &= \frac{P(E_1 E_2 E_3^c) + P(E_1 E_2^c E_3)}{P(E_1 E_2 E_3^c) + P(E_1 E_2^c E_3) + P(E_1^c E_2 E_3)} \\ &= \frac{\frac{2 \cdot 8 \cdot 3}{6 \cdot 12 \cdot 4} + \frac{2 \cdot 4 \cdot 1}{6 \cdot 12 \cdot 4}}{\frac{2 \cdot 8 \cdot 3}{6 \cdot 12 \cdot 4} + \frac{2 \cdot 4 \cdot 1}{6 \cdot 12 \cdot 4} + \frac{4 \cdot 8 \cdot 1}{6 \cdot 12 \cdot 4}} \\ &= \frac{7}{11}. \end{aligned}$$

Problem 10 For $i = 1, 2, 3$, let E_i be the event that the i -th card is a spade. Then

$$P(E_1 E_2 E_3) = \frac{13}{52} \frac{12}{51} \frac{11}{50}$$

and

$$P(E_2E_3) = P(E_1E_2E_3) + P(E_1^cE_2E_3) = \frac{13}{52} \frac{12}{51} \frac{11}{50} + \frac{39}{52} \frac{13}{51} \frac{12}{50}.$$

Thus

$$P(E_1|E_2E_3) = \frac{P(E_1E_2E_3)}{P(E_2E_3)} = \frac{11}{50}.$$

Problem 20 (a) $P(F|C) = \frac{P(FC)}{P(C)} = \frac{.02}{.05} = \frac{2}{5}.$

(b) $P(C|F) = \frac{P(FC)}{P(F)} = \frac{.02}{.52} = \frac{1}{26}.$

Problem 23 (a) Let W be the event that the ball selected urn II is white, E be the event that the transferred ball is white and F be the event that the transferred ball is red, then

$$P(W) = P(E)P(W|E) + P(F)P(W|F) = \frac{2}{3} \frac{1}{3} + \frac{1}{3} \frac{2}{3} = \frac{4}{9}.$$

(b)

$$P(E|W) = \frac{P(EW)}{P(W)} = \frac{P(E)P(W|E)}{P(W)} = \frac{1}{2}.$$

Problem 30 Let B and W be the events that the marble is black and white respectively, and let B_i be the event that box i is chosen. Then

$$P(B) = P(B_1)P(B|B_1) + P(B_2)P(B|B_2) = \frac{1}{2} \frac{1}{2} + \frac{1}{2} \frac{2}{3} = \frac{2}{12},$$

$$P(B_1|W) = \frac{P(B_1W)}{P(W)} = \frac{P(B_1)P(W|B_1)}{P(W)} = \frac{\frac{1}{2} \frac{1}{2}}{\frac{5}{12}} = \frac{3}{5}.$$

Problem 47 (a)

$$P(\text{all white}) = \frac{1}{6} \left(\frac{5}{15} + \frac{5}{15} \frac{4}{14} + \frac{5}{15} \frac{4}{14} \frac{3}{13} + \frac{5}{15} \frac{4}{14} \frac{3}{13} \frac{2}{12} + \frac{5}{15} \frac{4}{14} \frac{3}{13} \frac{2}{12} \frac{1}{11} \right).$$

(b)

$$P(3|\text{all white}) = \frac{\frac{1}{6} \frac{5}{15} \frac{4}{14} \frac{3}{13}}{P(\text{all white})}.$$

Problem 51 Let R be the event that she receives a job offer, S be the event that event of a strong recommendation, M the event of a moderate recommendation and W the event of a weak recommendation.

(a)

$$\begin{aligned} P(R) &= P(S)P(R|S) + P(M)P(R|M) + P(W)P(R|W) \\ &= (.8)(.7) + (.4)(.2) + (.1)(.1) = .65. \end{aligned}$$

(b)

$$P(S|R) = \frac{P(SR)}{P(R)} = \frac{P(S)P(R|S)}{P(R)} = \frac{(.8)(.7)}{.65} = \frac{56}{65}.$$

Similarly

$$P(M|R) = \frac{8}{65}, \quad P(W|R) = \frac{1}{65}.$$

Problem 56

$$P(\text{new}) = \sum_{i=1}^m p_i P(\text{new} | \text{the } n\text{-th coupon is of type } i) = \sum_{i=1}^m p_i (1-p_i)^{n-1}.$$

Problem 57 (a) $P(\text{original price after two days}) = \binom{2}{1} p(1-p) = 2p(1-p)$

(b) $P(\text{increase by one after three days}) = \binom{3}{2} p^2(1-p) = 3p^2(1-p)$

(c) $P(\text{increase on first day} | \text{increase by one after three days}) = \frac{p \cdot 2p(1-p)}{3p^2(1-p)} = \frac{2}{3}$

Problem 59 (a) $P(HHHH) = p^4$

(b) $P(THHH) = p^3(1-p)$

(c) The pattern $HHHH$ can *only* occur before $THHH$ if the first four coin flips come up heads. Hence, $P(THHH \text{ occurs before } HHHH) = 1 - p^4$.