Homework 3 — Solutions
Chapter 2

Problem 7  (a) There are $6^{15}$ outcomes in the sample space.

(b) There are $3^{15}$ outcomes without any blue-collar workers, so that there are $6^{15} - 3^{15}$ outcomes with at least one blue-collar worker.

(c) If there are no independents, then for each player, there are 4 outcomes, so that there are $4^{15}$ outcomes altogether.

Problem 8 Suppose that $A, B$ are mutually exclusive events for which $P(A) = 0.3$ and $P(B) = 0.5$.

(a) $P$(either $A$ or $B$) = $P(A) + P(B) = 0.8$ because $A$ and $B$ are mutually exclusive.

(b) $P(A$ occurs but $B$ does not) = 0.3.

(c) $P(A \cap B) = 0$.

Problem 9 Let $A$ be the event that a randomly chosen customer carries an Amex card, and let $V$ be the event that a randomly chosen customer carries a Visa card. Then $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(AB) = 0.24 + 0.61 - 0.11 = 0.74$. Hence, 74 percent of all customers carry an acceptable credit card.

Problem 12 Let $E$ be the event that a randomly chosen student is taking Spanish, let $F$ be the event that a randomly chosen student is taking French, and let $D$ be the event that a randomly chosen student is taking German.
Note that

\[
\begin{align*}
P(E) &= \frac{28}{100} = \frac{7}{25} \\
P(F) &= \frac{26}{100} = \frac{13}{50} \\
P(D) &= \frac{16}{100} = \frac{4}{25} \\
P(EF) &= \frac{12}{100} = \frac{3}{25} \\
P(ED) &= \frac{4}{100} = \frac{1}{25} \\
P(FD) &= \frac{6}{100} = \frac{3}{50} \\
P(EFD) &= \frac{2}{100} = \frac{1}{50}
\end{align*}
\]

(a) The probability of a student not being in any of these classes is

\[
1 - P(E \cup F \cup D) = 1 - (P(E) + P(F) + P(D) - P(EF) - P(ED) - P(FD) + P(EFD))
= 1 - \frac{28 + 26 + 16 - 12 - 4 - 6 + 2}{100}
= 1 - \frac{1}{2} = \frac{1}{2}.
\]

(b) We have

\[
\begin{align*}
P(\text{only } E) &= P(E) - P(EF) - P(ED) + P(EFD) = 0.14 \\
P(\text{only } F) &= P(F) - P(EF) - P(FD) + P(EFD) = 0.1 \\
P(\text{only } D) &= P(D) - P(ED) - P(FD) + P(EFD) = 0.08
\end{align*}
\]

Hence, \(P(\text{exactly one language class}) = 0.14 + 0.1 + 0.08 = 0.32\).

(c) Since fifty students are not taking any of the courses, the probability that neither of two randomly picked students is

\[
\frac{\binom{50}{2}}{\binom{100}{2}} = \frac{49}{198},
\]

so that the probability of at least one of them taking a language class is

\[1 - \frac{49}{198} = \frac{149}{198}\]

Problem 17 There are \(64 \cdot 63 \cdot 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57\) ways of arranging 8 castles on a chess board. Of these, there are \(64 \cdot 49 \cdot 36 \cdot 25 \cdot 16 \cdot 9 \cdot 4 \cdot 1 = \prod_{i=1}^{8} i^2\) in
which none of the rooks can capture any of the others. So the answer is

\[
\prod_{i=1}^{8} i^2 / 64 \cdot 63 \cdot 62 \cdot 61 \cdot 60 \cdot 59 \cdot 58 \cdot 57 = 171
\]

Problem 18

\[
\frac{2 \cdot 4 \cdot 16}{52 \cdot 51}
\]

Problem 20 Let \( A \) be the event that you are dealt a blackjack, and let \( B \) be the event that the dealer is dealt a blackjack.

Then

\[
P(A) = P(B) = \frac{2 \cdot 4 \cdot 16}{52 \cdot 51}
\]

\[
P(AB) = \frac{4 \cdot 4 \cdot 16 \cdot 3 \cdot 15}{52 \cdot 51 \cdot 50 \cdot 49}
\]

\[
P(A \cup B) = P(A) + P(B) - P(AB) = 0.0948.
\]

Hence, the probability that neither you nor the dealer is dealt a blackjack is \( 1 - P(A \cup B) = 0.9052 \).

Problem 21 (a) \( P(1) = \frac{4}{20} = \frac{1}{5}, \) \( P(2) = \frac{8}{20} = \frac{2}{5}, \) \( P(3) = \frac{5}{20} = \frac{1}{4}, \) \( P(4) = \frac{2}{20} = \frac{1}{10}, \) and \( P(5) = \frac{1}{20} \).

(b) There are 48 children altogether, so that \( P(1) = \frac{4}{48} = \frac{1}{12}, \) \( P(2) = \frac{8}{48} = \frac{1}{6}, \) \( P(3) = \frac{5}{48} = \frac{5}{16}, \) \( P(4) = \frac{2}{48} = \frac{1}{8}, \) and \( P(5) = \frac{1}{24} \).

Problem 25 Let \( E_n \) be the event that a sum of 5 occurs on the \( n \)th roll, and no sum of 5 or 7 occurs on the first \( n - 1 \) rolls. There are 36 outcomes of a single roll, and four of them give a sum of 5, while 6 of them give a sum of 7. Hence,

\[
P(E_n) = \left( \frac{26}{36} \right)^{n-1} \left( \frac{4}{36} \right) = \left( \frac{13}{18} \right)^{n-1} \frac{1}{9}.
\]

A sum of 5 occurs before a sum of 7 precisely if the events \( E_n \) occurs for some \( n \). Since \( E_n \) and \( E_m \) are disjoint if \( n \neq m \), the desired probability is

\[
\sum_{n=1}^{\infty} P(E_n) = \sum_{n=1}^{\infty} \left( \frac{13}{18} \right)^{n-1} \cdot \frac{1}{9} = \frac{1}{9} \cdot \frac{1}{1 - \frac{13}{18}} = \frac{118}{9} = \frac{2}{5}.
\]
Problem 27

\[ P(A \text{ wins in one move}) = \frac{3}{10} \]

\[ P(A \text{ wins in three moves}) = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{3}{8} = \frac{7}{40} \]

\[ P(A \text{ wins in five moves}) = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} = \frac{1}{12} \]

\[ P(A \text{ wins in seven moves}) = \frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} \cdot \frac{4}{7} \cdot \frac{3}{6} \cdot \frac{2}{5} \cdot \frac{4}{3} = \frac{1}{40} \]

\[ P(A \text{ wins}) = \frac{3}{10} + \frac{7}{40} + \frac{1}{12} + \frac{1}{40} = \frac{7}{12} \]

Problem 28 (a) Without replacement:

\[ P(\text{all three balls are the same color}) = \frac{\binom{5}{3} + \binom{6}{3} + \binom{8}{3}}{\binom{19}{3}} \]

With replacement:

\[ P(\text{all three balls are the same color}) = \left( \frac{5}{19} \right)^3 + \left( \frac{6}{19} \right)^3 + \left( \frac{8}{19} \right)^3 \]

(b) Without replacement:

\[ P(\text{all three balls are of different colors}) = \frac{\binom{5}{1} \cdot \binom{6}{1} \cdot \binom{8}{1}}{\binom{19}{3}} \]

With replacement:

\[ P(\text{all three balls are of different colors}) = 3! \cdot \frac{5}{19} \cdot \frac{6}{19} \cdot \frac{8}{19} \]

Problem 32 There are \((b+g)!\) ways to line up the children. There are \(g \cdot (b+g-1)!\) arrangements with a girl in the \(i\)th position. The desired probability is

\[ \frac{g(b+g-1)!}{(b+g)!} = \frac{g}{b+g} \]

Problem 37 (a) There are \(\binom{10}{5}\) selections for the final exam. The number of selections that allow the student to solve all problems is \(\binom{7}{3}\), so that the desired probability is \(\frac{\binom{7}{3}}{\binom{10}{5}} = 0.08333\).
(b) There are \( \binom{7}{4} \cdot \binom{3}{1} \) selections that’ll let the student solve exactly four problems, so that the probability of solving at least four problems is \( \frac{\binom{7}{4}+\binom{3}{1}\cdot\binom{1}{1}}{\binom{10}{5}} = \frac{1}{2}. \)