Wk 12
Wed
p.1

§7.2 Expectation of a function of joint RVs:

Definitions: discrete: $\mathbb{E}[g(X,Y)] = \sum_{\text{all } x}\sum_{\text{all } y} g(x,y)\,p(x,y)$
where $p(x,y)$ is jt pmf of (X,Y).

Continuous: $\mathbb{E}[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\,\underbrace{f(x,y)}_{\text{jt pdf of }(X,Y).}\,dx\,dy$

Example: If X & Y are independent Uniform(0,L) RVs, what is the expected distance between them?
Need to compute $\mathbb{E}[|X-Y|]$, i.e., $g(X,Y) = |X-Y|$.

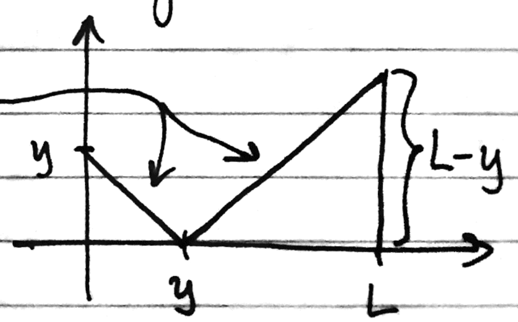
Joint pdf is $f(x,y) = \begin{cases} 1/L^2, & \text{if } x,y \in (0,L), \\ 0, & \text{else.} \end{cases}$

So:

$$\mathbb{E}[|X-Y|] = \iint_{\mathbb{R}^2} |x-y|\,f(x,y)\,dx\,dy$$
$$= \int_0^L\int_0^L |x-y|\cdot L^{-2}\,dx\,dy$$

Fix $y \in (0,L)$ and focus on the x-integral

$$\int_0^L |x-y|\,dx = \text{area under "curve"}$$
$$= \frac{1}{2}y^2 + \frac{1}{2}(L-y)^2$$
$$= \left(\frac{1}{2}L^2 - Ly + y^2\right)$$



Thus

$$\mathbb{E}[|X-Y|] = L^{-2}\int_0^L \left(\frac{1}{2}L^2 - Ly + y^2\right) dy$$
$$= L^{-2}\left[\frac{1}{2}L^2 y - \frac{1}{2}Ly^2 + \frac{1}{3}y^3\right]_{y=0}^{L} = \frac{L}{3}.$$