End of Ch. 5 in Ross

Another example of figuring out the distribution of a function of a RV:

Say $X \sim \text{Uniform}(-1,1)$. What is $Y = |X|$?

Cdf F_Y first:

$F_Y(y) = \mathbb{P}(|X| \leq y)$ and note that

if $y<0$, $F_Y(y) = 0$ and if $y \geq 1$, $F_Y(y) = 1$.

For $y \in [0,1]$,

$$F_Y(y) = \mathbb{P}(-y \leq X \leq y) = F_X(y) - F_X(-y)$$

Second, take the derivative to get pdf f_Y:

$$f_Y(y) = F_Y'(y) = F_X'(y) - F_X'(-y)$$

$$= f_X(y) + f_X(-y)$$

And the pdf of X is $f_X(x) = \begin{cases} \frac{1}{2}, & \text{if } -1 \leq x \leq 1 \\ 0, & \text{else}, \end{cases}$

so $f_Y(y) = \frac{1}{2} + \frac{1}{2} = 1$ for $y \in [0,1]$.

We recognize $Y = |X|$ as a Uniform(0,1) RV.

Note: $f_Y \neq |f_X|$, just like $f_{g(X)} \neq g(f_X)$.

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Chapter 6 Joint RVs

If 2 or more RVs might depend on each other, we study them by their joint pmf/pdf/cdf.

The joint cdf is the same for discrete & continuous:

$$F(a,b) := \mathbb{P}(X \le a, Y \le b) \text{ for pair } (X,Y).$$

We can get individual info about X or Y by

marginal cdf for X: $F_X(a) = \mathbb{P}(X \le a)$
$= \mathbb{P}(X \le a, Y < \infty)$
$= F(a, \infty)$

&

marginal cdf for Y:

$$F_Y(b) = F(\infty, b).$$

Note: $F(\infty,\infty) = 1$, but often $F(a,\infty) \neq 1$.

Common mistake: $\mathbb{P}(X > a, Y > b) = 1 - \mathbb{P}(X \le a, Y \le b)$
wrong

The correct version uses Inclusion-Exclusion:

$\mathbb{P}(X > a, Y > b) = 1 - \mathbb{P}(\{X > a, Y > b\}^c)$

DeMorgan: $= 1 - \mathbb{P}(\{X > a\}^c \cup \{Y > b\}^c)$

$= 1 - \left(\mathbb{P}(X \le a) + \mathbb{P}(Y \le b) - \mathbb{P}(\{X \le a\} \cap \{Y \le b\})\right)$

$= 1 - F_X(a) - F_Y(b) + F(a,b)$