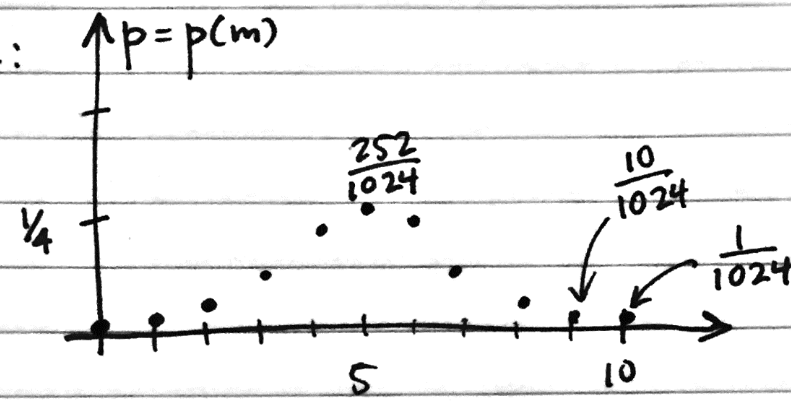
Wk 5
Fri
p.1

(Binomial recursion formula proved in §4.6.1)

Example: 10 fair coins, $X = \#\text{Heads}$.

pmf is $p(m) = \binom{10}{m} \cdot \left(\frac{1}{2}\right)^{10} = \dfrac{\binom{10}{m}}{1024}$

Pic:



This is a first glimpse of the Central Limit Theorem — $p(m)$ looks a bit Gaussian-ish.

§4.7 **Poisson RVs** model # of events per unit time,
e.g.,
- α-particle emission from radioactive material
- arrivals in a queue
- mutations in a segment of DNA per unit of radiation.

$X \sim \text{Poisson}(\lambda)$ has pmf

$$p(n) = \mathbb{P}(X=n) := e^{-\lambda} \frac{\lambda^n}{n!}, \quad \text{for } n = 0, 1, 2, \ldots$$

and $\lambda > 0$ is a parameter, called the rate.

Does this pmf sum to 1?

$$\sum_{n=0}^{\infty} p(n) = \sum_{n \geq 0} e^{-\lambda} \frac{\lambda^n}{n!} = e^{-\lambda} \underbrace{\sum_{n \geq 0} \frac{\lambda^n}{n!}}_{\text{Taylor series for } e^{\lambda}}$$

$$= e^{-\lambda} \cdot e^{\lambda} = 1. \text{ Yes.}$$

Wk 5
Fri
p. 2

Ex: On average, 3.2 α-particles are emitted per sec. by a sample of uranium. What is the probability that no more than 2 α-particles are emitted in 1 second?

Rate: $\lambda = 3.2$, $X \sim \text{Poisson}(3.2)$

We want to compute:

$$\mathbb{P}(X \leq 2) = p(0) + p(1) + p(2)$$

$$= e^{-\lambda}\frac{\lambda^0}{0!} + e^{-\lambda}\frac{\lambda^1}{1!} + e^{-\lambda}\frac{\lambda^2}{2!}$$

$$= e^{-3.2}\left(1 + 3.2 + \frac{3.2^2}{2}\right) \underset{\text{W/A}}{\cong} 0.38$$

estimate: $\cong \frac{1}{27}\left(4.2 + \frac{10}{2}\right) = \frac{9.2}{27} \cong \frac{1}{3}$, close to

What is the mean of $X \sim \text{Poisson}(\lambda)$?

$$\mu = E[X] = \sum_{n=0}^{\infty} n \cdot e^{-\lambda}\frac{\lambda^n}{n!} = e^{-\lambda}\sum_{n=1}^{\infty}\frac{\lambda^n}{(n-1)!}$$

$$= \lambda e^{-\lambda}\sum_{n \geq 1}\frac{\lambda^{n-1}}{(n-1)!}$$

change vars: $j = n-1$

$$= \lambda e^{-\lambda}\sum_{j \geq 0}\frac{\lambda^j}{j!}$$

$$= \lambda e^{-\lambda} \cdot e^{\lambda}$$

$$\mathbb{E}[\text{Poisson}(\lambda)] = \lambda.$$

(Quiz)