ADDENDUM TO: "STALLINGS FOLDINGS AND SUBGROUPS OF FREE GROUPS", JOURNAL OF ALGEBRA 248 (2002)

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We would like to acknowledge some important work relevant to our article [9], of which we were not aware at the time of writing that paper. Theorem 13.1 of [9] provides an algorithm for recognizing whether a finite set of elements in a free group F_n generates an isolated (also referred to as *pure*) subgroup of F_n. This result was first proved by J.-C. Birget, S. Margolis, J. Meakin and P. Weil in [2] using the theory of inverse monoids. Moreover, in Remark 13.2 of [9] we noted that the algorithm provided by our Theorem 13.1 was one of the few algorithms in the paper with high complexity. We also asked if a substantially easier algorithm existed. It turns out that in [2, 3] J.-C. Birget, S. Margolis, J. Meakin and P. Weil proved that in fact the problem of deciding if a finitely generated subgroup of a free group is isolated is PSPACE-complete and thus is intrinsically difficult.

We would also like to mention that concepts equivalent to our notion of a "principal quotient" [9] also appeared in the work of S. Margolis, M. Sapir and P. Weil [11] as "overgroups of a subgroup" and in the work of E.Ventura [13] (see also [5]) as "fringes of subgroups".

Some other results related to the issues explored in [9] also appear in [1, 4, 6, 7, 10, 12, 14] and other sources.

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