1. Let $w$ be a 2-form on $\mathbb{R}^3$ given by
   
   \[ w = 2xy \, dx \wedge dy - (x^2 + y + 1) \, dy \wedge dz \]
   
   a) Find $dw$

   b) Let $X = 2xy \frac{\partial}{\partial x} + (z^2 + x^2) \frac{\partial}{\partial z}$.

   Compute the 1-form $i_X(w)$

   c) Let $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be given by

   \[ f(u, v) = (u + v, u^2 + 1, 3uv - 4) \]

   Compute the form $f^*w$ on $\mathbb{R}^2$

   d) Let $\eta = dx + dy + 3dz$. Compute $w \wedge \eta$.

2. Consider the following vector fields on $\mathbb{R}^3$

   \[ X = 2yx \frac{\partial}{\partial x} + (z^2 + x^2) \frac{\partial}{\partial z} \]

   \[ Y = e^{xy} \frac{\partial}{\partial x} + \cos(2xz) \frac{\partial}{\partial y} \]

   Compute the vector field $[X, Y]$ on $\mathbb{R}^3$
3. Let $X, Y, Z$ be smooth vector fields on $M$.
Prove the Jacobi identity:

$$[[X,Y],Z] + [[Y,Z],X] + [[Z,X],Y] = 0$$

4. Let $W$ be an $r$-form on a vector space $V$, where $r \geq 2$.
Let $v_1, v_2, \ldots, v_r \in V$ be linearly dependent vectors.
Prove that $W(v_1, v_2, \ldots, v_r) = 0$

5. Let $W = \alpha(x,y,z) \, dx \wedge dy + \beta(x,y,z) \, dx \wedge dz + \\ + \gamma(x,y,z) \, dy \wedge dz$

a) Write an explicit condition for $\alpha, \beta, \gamma$
equivalent to the condition $\text{d}W = 0$.

b) Write an explicit condition for $\alpha, \beta, \gamma$
equivalent to saying that $W$ is exact.

c) Prove that

$$W = z^2 \, dx \wedge dy + 2zy \, dx \wedge dz$$
is an exact 2-form on $\mathbb{R}^3$. 