Quiz 5 (Solutions); Friday, March 6, 2009

1. For each of the following statements indicate if it is true or false. You do not need to explain your answers here.
   (1) The element $18 \in \mathbb{Z}_{20}$ is a generator of $\mathbb{Z}_{20}$.
   (2) If $g \in G$ and $|g| = 36$ then $|g^4| = 4$.
   (3) If $G$ is a cyclic group then every subgroup of $G$ is abelian.
   (4) If $a, b \in G$ are such that $|\langle a \rangle| = |\langle b \rangle|$ then $a^{|b|} = 1_G$.
   (5) If $G = \langle a \rangle$ is a cyclic group generated by $a$ then $\langle a^5, a^4 \rangle = G$.

Answers:
   (1) False. Since $gcd(18, 20) = 2 \neq 1$, it follows that $18$ is not a generator of $\mathbb{Z}_{20}$.
   (2) False. We have $|g^4| = \frac{36}{4} = 9$.
   (3) True. A cyclic group is abelian and every subgroup of an abelian group is abelian.
   (4) True. We know that the order of a cyclic group generated by an element is equal to the order of that element. Therefore $a$ and $b$ have equal orders and hence $a^{|b|} = 1_G$.
   (5) True. We have $a^5 \cdot a^{-4} = a \in \langle a^5, a^4 \rangle$ and so $G = \langle a \rangle \subseteq \langle a^5, a^4 \rangle \subseteq G$. Therefore $G = \langle a^5, a^4 \rangle$. 
