Quiz 1 (Solution); Friday, January 29

For each of the following statements indicate if it is true or false. You do not need to provide explanations for your answers.

1. Let $a, b, x, y, d$ be integers, where $d \geq 1$, such that $ax + by = d$. Then $\gcd(a, b) = d$.

2. The number $2^{100}$ has exactly 100 distinct positive divisors.

3. If $p$ and $q$ are distinct primes then there exist integers $x$ and $y$ such that $xp + yq = 1$.

4. If an integer $n \geq 2$ has no prime divisors $p$ such that $p \leq \sqrt{n}$ then $n$ is prime.

Solution:

1. FALSE. For example, $1 \cdot 5 + 1 \cdot 7 = 12$ but $\gcd(1, 1) = 1 \neq 12$.

2. FALSE. The number $2^{100}$ has exactly 101 distinct positive divisors: $1, 2, 2^2, 2^4, \ldots, 2^{100}$.

3. TRUE.

4. TRUE.