Quiz 10 (Solutions); Friday, April 24, 2009

1. For each of the following statements indicate if it is true or false. You do not need to explain your answers here.

   (1) If $R$ is a commutative ring and $A \subseteq R$ is a maximal ideal then $R/A$ is an integral domain.
   (2) If $R$ is a commutative ring and $A \subseteq R$ is an ideal then $\text{char}(R/A) = \text{char}(R)$.
   (3) If $R$ is a field then the set of all ideals in $R$ has at most five elements.
   (4) The map $f : \mathbb{C} \to \mathbb{R}$ defined by $f(x + iy) = x + y$ (where $x, y \in \mathbb{R}$) is a ring homomorphism.
   (5) If $R \neq 0$ is any ring, then there exists exactly one ring homomorphism $\mathbb{Z} \to R$.

Answers:

   (1) True. If $A$ is a maximal ideal then $R/A$ is a field and hence an integral domain.
   (2) False. For example, $\text{char}(\mathbb{Z}) = 0$ but for $m \geq 2$ we have $\text{char}(\mathbb{Z}/m\mathbb{Z}) = \text{char}(\mathbb{Z}_m) = m \neq 0$.
   (3) True. A field $R$ has only two ideals, namely $R$ and $\{0\}$, and it is true that $2 \leq 5$.
   (4) False.
   (5) True. If $\phi : \mathbb{Z} \to R$ is a ring homomorphism then $\phi(1) = 1_R$ and hence $\phi(n) = n \cdot 1_R$ for every $n \in \mathbb{Z}$. Also, the function $\phi : \mathbb{Z} \to R$ given by $\phi(n) = n \cdot 1_R$, where $n \in \mathbb{Z}$, is a ring homomorphism.