Quiz 11 (Solution); Friday, April 29, 2005

For each of the following subsets determine whether it is an ideal in the given ring. If not, explain why not.

1. \{f(x) \in \mathbb{Z}[x] : f(0) = 0\} \subseteq \mathbb{Z}[x];

2. \{a + bi | a, b \in \mathbb{Q}\} \subseteq \mathbb{C};

3. \{[0]_6, [3]_6\} \subseteq \mathbb{Z}/6\mathbb{Z}.

Solution.

(1) This is an ideal in \(\mathbb{Z}[x]\). In fact, one can check that it is a principal ideal:

\[ \{f(x) \in \mathbb{Z}[x] : f(0) = 0\} = (x) \triangleleft \mathbb{Z}[x]; \]

(2) The set \(I = \{a + bi | a, b \in \mathbb{Q}\}\) is not an ideal in \(\mathbb{C}\).

Indeed, \(1 = 1 + 0 \cdot i \in I\) but \(\sqrt{2} \cdot 1 = \sqrt{2} \notin I\).

(3) Yes, this is an ideal in \(\mathbb{Z}/6\mathbb{Z}\) and in fact, this is a principal ideal:

\[ \{[0]_6, [3]_6\} = ([3]_6) \triangleleft \mathbb{Z}/6\mathbb{Z}. \]