H/wk 6, Hint for Problem 2.54

(i) Prove that if \( g, f \in \bigcap_{i \in I} x_i S_i \) then
\[
g^{-1}f \in \bigcap_{i \in I} S_i
\]

(ii) Use induction on the number of distinct subgroups among \( S_1, \ldots, S_n \) (note that this is not the same thing as induction on \( n \)).

In the inductive step you will need to argue that if \( S \in \{ S_1, \ldots, S_n \} \) has infinite index and \( G = \bigcup_{i=1}^n x_i S_i \) then \( S \) is contained in the union of finitely many cosets of the subgroups from \( \{ S_1, \ldots, S_n \} - \{ S \} \).