Pre-class worksheet due Friday, September 9 (Solutions).

1. Write down the matrix of a linear trasformation \( T : \mathbb{R}^2 \to \mathbb{R}^3 \) given by the formula
\[
 f([x, y]^T) = [2x - y, x + 5y, x]^T.
\]
**Answer:** We have:
\[
 A_T = \begin{bmatrix} 2 & -1 \\ 1 & 5 \\ 1 & 0 \end{bmatrix}.
\]

2. Let \( T_1 : V \to W \) and \( T_2 : W \to U \) be \( \mathbb{R} \)-linear maps. Does it follow that \( T_2 \circ T_1 : V \to U \) is again \( \mathbb{R} \)-linear?

**Solution:**
Yes, \( T_2 \circ T_1 : V \to U \) is again \( \mathbb{R} \)-linear.
Indeed, by linearity of both \( T_1 \) and \( T_2 \), for any \( \vec{v}_1, \vec{v}_2 \in V \) we have
\[
 (T_2 \circ T_1)(\vec{v}_1 + \vec{v}_2) = T_2(T_1(\vec{v}_1 + \vec{v}_2)) = T_2(T_1(\vec{v}_1) + T_2(\vec{v}_2)) = \\
 = T_2(T_1(\vec{v}_1)) + T_2(T_1(\vec{v}_2)) = (T_2 \circ T_1)(\vec{v}_1) + (T_2 \circ T_1)(\vec{v}_2).
\]
Similarly, for all \( \vec{v} \in V \) and \( \alpha \in \mathbb{R} \) we have
\[
 (T_2 \circ T_1)(\alpha \vec{v}) = T_2(T_1(\alpha \vec{v})) = T_2(\alpha T_1(\vec{v})) = \alpha T_2(T_1(\vec{v})) = \alpha(T_2 \circ T_1)(\vec{v}).
\]
Thus the map \( T_2 \circ T_1 : V \to U \) is \( \mathbb{R} \)-linear.

3. Is true that \( \text{trace}(AB) = \text{trace}(A)\text{trace}(B) \) for all \( A, B \in M_{2,2}(\mathbb{R}) \)?

**Solution.** No, this statement is false. For example, for \( A = B = I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) we have \( AB = I_2 \cdot I_2 = I_2 \). Thus \( \text{trace}(A) = \text{trace}(B) = \text{trace}(AB) = 1 + 1 = 2 \). However \( \text{trace}(AB) = 2 \neq 4 = 2 \cdot 2 = \text{trace}(A)\text{trace}(B) \).