Problem 1.
Find the general solution of the following differential equation on the interval \( x > 0 \):

\[
\frac{dy}{dx} + \frac{2y}{x} = \frac{2}{x^3 + x}.
\]

Solution.
This is a linear first-order differential equation. First, we compute the integrating factor:

\[
\rho(x) = e^{\int \frac{2}{x} \, dx} = e^{2 \ln |x|} = e^{\ln(x^2)} = x^2.
\]

We now multiply the original equation by \( \rho(x) = x^2 \) and get:

\[
\frac{dy}{dx} x^2 + 2yx = \frac{2x^2}{x^3 + x} = \frac{2x}{x^2 + 1}
\]

\[
\frac{d}{dx}(yx^2) = \frac{2x}{x^2 + 1}
\]

\[
yx^2 = \int \frac{2x}{x^2 + 1} \, dx = \int \frac{d(x^2 + 1)}{x^2 + 1} = \ln |x^2 + 1| + C = \ln(x^2 + 1) + C
\]

\[
y = \frac{\ln(x^2 + 1)}{x^2} + \frac{C}{x^2}.
\]

Thus the general solution of our equation on the interval \( x > 0 \) is

\[
y = \frac{\ln(x^2 + 1)}{x^2} + \frac{C}{x^2}
\]

where \( C \in \mathbb{R} \) is an arbitrary constant.