Problem 1.
Consider the following wave equation problem

\[
\begin{cases}
y_{tt} = 9y_{xx}, & 0 < x < 2, \ t > 0 \\
y(0, t) = y(2, t) = 0, & t > 0 \\
y(x, 0) = 4x + 5, & 0 < x < 2 \\
y_t(x, 0) = 0, & 0 < x < 2.
\end{cases}
\]

Find the precise numerical value (NOT an infinite sum expression!) for \(y(0.5, 13)\).

Solution.
We have \(a = \sqrt{9} = 3\) and \(L = 2\).

We will use D’Alambert’s form of the solution for this problem:

\[y(x, t) = \frac{1}{2}[f_0(x + at) + f_0(x - at)]\]

where \(f_0(x)\) is the 4-periodic odd extension of \(f(x) = 3x + 5\) \(0 < x < 2\).

Thus

\[y(0.5, 13) = \frac{1}{2}[f_0(0.5 + 3 \cdot 13) + f_0(0.5 - 3 \cdot 13)] = \]

\[= \frac{1}{2}[f_0(39.5) + f_0(-38.5)] = \frac{1}{2}[f_0(-0.5) + f_0(1.5)] = \]

\[= \frac{1}{2}[-f(0.5) + f(1.5)] = \frac{1}{2}[-7 + 11] = 2.\]