Diagnostic Quiz 11. October 10, 2017

1.
For each of the following statements indicate whether it is true or false.
(a) For every vectors $\vec{v}_1, \vec{v}_2 \in \mathbb{R}^3$ the $3 \times 3$ matrix

$$A = [\vec{v}_1 | \vec{v}_2 | \vec{v}_1 + \vec{v}_2]$$

is not invertible.

Answer: True.
Indeed, the columns of $A$ are linearly dependent since if we denote $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$ then $\vec{v}_1 + \vec{v}_2 - \vec{v}_3 = \vec{0}$. Hence by part (e) of Theorem 8 on p. 114 the matrix $A$ is not invertible.

(b) If $A, B, C$ are $n \times n$ matrices such that $A$ is invertible and $AB = AC$ then $B = C$.

Answer: True.
Indeed, multiply the equality $AB = AC$ on the left by $A^{-1}$. Then we get
$$B = I_n B = (A^{-1} A) B = A^{-1} (AB) = A^{-1} (AC) = (A^{-1} A) C = I_n C = C.$$

(c) If $A$ is an $n \times n$ with linearly independent rows then $A$ is invertible.
Answer: True.
Indeed, consider the matrix $A^T$. Then the rows of $A$ become columns of $A^T$. The assumption on $A$ means that the columns of $A^T$ are linearly independent and hence $A^T$ is invertible by part (e) of of Theorem 8 on p. 114. Therefore the matrix $A$ is invertible by part (l) of Theorem 8 on p. 114.