Let $f : \mathbb{R} \to \mathbb{R}$ be given by $f(x) = e^x - 1$.

1. Is the function $f$ one-to-one?
2. Find the range of $f$.
3. Is $f$ surjective?
4. For $g : \mathbb{R} \to \mathbb{R}$, $g(x) = x^2 + x$, compute the functions $f \circ g$ and $g \circ f$.

**Solution.**

1. The function $f(x)$ is one-to-one. Indeed, if $f(x_1) = f(x_2)$ for some $x_1 = x^2 \in \mathbb{R}$ then $e^{x_1} - 1 = e^{x_2} - 1$. Hence $e^{x_1} = e^{x_2}$ and $x_1 = x_2$.

2. The range of $f$ is $(-1, \infty) = \{ y \in \mathbb{R} \mid y > -1 \}$.
   Indeed, by definition,
   \[
   \text{range}(f) = \{ f(x) \mid x \in \mathbb{R} \} = \{ e^x - 1 \mid x \in \mathbb{R} \} = \{ y \in \mathbb{R} \mid y = e^x - 1 \text{ for some } x \in \mathbb{R} \} = \{ y \in \mathbb{R} \mid y - 1 = e^x \text{ for some } x \in \mathbb{R} \} = \{ y \in \mathbb{R} \mid y - 1 > 0 \} = \{ y \in \mathbb{R} \mid y > -1 \}.
   \]

3. No, the function $f$ is not surjective. For example, the number $-3$ belongs to the co-domain of $f$ (which, by definition of $f$, is the set $\mathbb{R}$), but $-3 \notin (-1, \infty) = \text{range}(f)$.

4. We have
   \[
   (f \circ g)(x) = f(g(x)) = f(x^2 + x) = e^{x^2 + x} - 1,
   \]
   \[
   (g \circ f)(x) = g(f(x)) = g(e^x - 1) = (e^x - 1)^2 + e^x - 1 = e^{2x} - e^x.
   \]