1. Using induction, prove that for every integer \( n \geq 2 \)

\[ 3^n \geq 2n^2 + 1. \]

**Solution. 1) Base of Induction.** First, check if the statement \( 3^n \geq 2n^2 + 1 \) holds for \( n = 2 \).

We have \( 3^2 = 9 \) and \( 2 \cdot 2^2 + 1 = 2 \cdot 4 + 1 = 9 \). Since \( 9 \geq 9 \), the required statement does hold for \( n = 2 \).

**2) Inductive Step.**

Let \( k \geq 2 \) and suppose that \( 3^k \geq 2k^2 + 1 \) is known to hold. We need to derive that \( 3^{k+1} \geq 2(k+1)^2 + 1 \), that is, \( 3^{k+1} \geq 2(k^2 + 2k + 1) + 1 \), that is, \( 3^{k+1} \geq 2k^2 + 4k + 3 \).

The inductive hypothesis \( 3^k \geq 2k^2 + 1 \) by multiplying by 3 implies

\[ 3^{k+1} \geq 6k^2 + 3. \]

To show that \( 3^{k+1} \geq 2k^2 + 4k + 3 \) it suffices to establish that for \( k \geq 2 \) we have \( 6k^2 + 3 \geq 2k^2 + 4k + 3 \).

We have:

\[ 6k^2 + 3 \geq 2k^2 + 4k + 3 \]

is equivalent to:

\[ 4k^2 \geq 4k \]

by dividing by \( 4k > 0 \), is equivalent to:

\[ k \geq 1, \]

which holds since by assumption \( k \geq 2 \). Thus for \( k \geq 2 \) we have \( 6k^2 + 3 \geq 2k^2 + 4k + 3 \), as required.