1. Let \( \lambda, \alpha \) be complex numbers. Solve by any valid method the ODE
\[
(D - \lambda)f(t) = e^{\alpha t}.
\]
Note that the answer changes when \( \lambda = \alpha \).

2. Let \( V \) be the vector space of all polynomials (in one variable) with real (or complex) coefficients. Define a linear map \( T : V \to V \) by \( T(p) = p' - 3p \).
Find the null space \( \mathcal{N}(T) \) and the range \( \mathcal{R}(T) \). Comment: In finding these spaces you must find polynomial solutions to the appropriate equations.

3. Let \( V \) be as in problem 2. Define \( T : V \to V \) by \( T(p) = p'' - 3p' + 2p \).
Let \( V \) be the space of all infinitely differentiable functions on \( \mathbb{R} \). What is \( \mathcal{N}(T) \)?

4. Let \( M \) be the matrix
\[
\begin{pmatrix}
0 & 4 \\
1 & 2
\end{pmatrix}.
\]
Find the inverse of \( M \) using row operations; thus find EXPLICIT elementary row matrices \( E_j \) such that
\[
E_3E_2E_1M = I.
\]
Comment 1: Thus
\[
M = E_1^{-1}E_2^{-1}E_3^{-1}.
\]
Comment 2: Factoring two-by-two matrices into elementary row matrices is used in transmission lines and in optics, such as the design of lasers.

5. Decide whether the matrices are row equivalent:
\[
\begin{pmatrix}
0 & 0 \\
1 & 0
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
2 & 1 \\
0 & 0
\end{pmatrix}
\]
\[
\begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9
\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
0 & 0 & 0
\end{pmatrix}.
\]

6. Solve (use a computer if you wish) the linear system
\[
\begin{align*}
21x + 35y - 52z &= 243 \\
98x + 28y - 88z &= 380.
\end{align*}
\]
Hint: \((4, 9, 3)\) is a particular solution and \((4, 8, 7)\) is a solution to the homogeneous equation.