1. Consider the linear system

\[
\begin{align*}
  x + y + z &= 11 \\
  2x - y + z &= 13 \\
  y + 3z &= 19.
\end{align*}
\]

Convert it to matrix form, solve by row operations, and check your answer.

2. Consider the linear system (expressed in matrix form) of four equations in six unknowns.

\[
\begin{pmatrix}
  1 & 2 & 3 & 0 & 0 & 1 \\
  0 & 1 & 0 & 1 & 2 & 8 \\
  0 & 0 & 0 & 1 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 & 5
\end{pmatrix}
\]

Find the most general solution, and write it in the form

\[x = x_0 + \sum c_j v_j.\]

3. Do the same problem if the third row is replaced with

\[
\begin{pmatrix}
  0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\]

4. Let \(n\) be a positive integer. “The collection of real polynomials (in one variable) of degree \(n\)” does not form a vector space. Why not? Add two words to the phrase in quotes to make it a vector space.

5. Suppose \(A = (a_{jk})\) and \(B = (b_{kl})\) and \(C = (c_{lm})\) are matrices of real numbers for which \(AB\) is defined and \(BC\) is defined. Carefully write out the proof that \((AB)C = A(BC)\).

6. Compute the matrix product \(P^{-1}QP\) if

\[
Q = \begin{pmatrix}
  -1 & 8 \\
  -4 & 11
\end{pmatrix}, \quad
P = \begin{pmatrix}
  2 & 1 \\
  1 & 1
\end{pmatrix}.
\]

7. Let \(J : \mathbb{R}^2 \to \mathbb{R}^2\) be the linear mapping with \(J(x, y) = (-y, x)\).
   a. What does \(J\) do geometrically?
   b. Find \(J^2\). Explain how the answer fits with part a.
   c. Find invertible two-by-two real matrices \(A, B\) such that

\[
(A + B)^{-1} = A^{-1} + B^{-1}.
\]

8. The trace of a square \((n\text{-by-}n)\) matrix is the sum of the diagonal elements. This concept turns out to be of major importance in both theory and application.
   a) Note that \(\text{trace}(I) = n\).
   b) Prove that \(\text{trace}(A + B) = \text{trace}(A) + \text{trace}(B)\) and \(\text{trace}(cA) = c \text{trace}(A)\) for scalars \(c\).
   c) Prove that \(\text{trace}(AB) = \text{trace}(BA)\).
   d) Hard! Prove that trace is the unique map from the space of \(n\text{-by-}n\) matrices to \(\mathbb{R}\) satisfying these properties.