Lecture 42 Results for the quartic surface

$X_0 = \{ x_0^4 + x_1^4 + x_2^4 + x_3^4 = 0 \} \subset \mathbb{P}^3$

$X_\infty = \{ x_0 x_1 x_2 x_3 = 0 \}$ $\qquad B = X_0 \cap X_\infty$

$M_0 = X_0 \setminus B$ Last time we constructed 64 Lagrangian Spheres in M_0 as vanishing cycles of the "Dwork pencil"

$$X_z = \{ x_0^4 + x_1^4 + x_2^4 + x_3^4 - z\, x_0 x_1 x_2 x_3 = 0 \}$$

Let $\mathcal{F}_{64}$ denote the total morphism A_∞-algebra of these objects, as objects of the Affine Fukaya category $\mathcal{F}(M_0)$

Let $\mathcal{F}_{64,q}$ denote the total morphism A_∞-algebra of the same objects, as objects of the relative Fukaya category $\mathcal{F}(X_0, B)$.

Let $\mathcal{F}^*_{64,q} = \mathcal{F}_{64,q} \otimes_{\Lambda_N} \Lambda_{\mathbb{Q}}$

We will now sketch out the proof of HMS for X_0

$H^0(\mathcal{F}(X_0)^{perf}) \cong \psi^* D^b Coh(Z_q^*)$. There are a number of things to establish

Lemma 1 The 64 vanishing cycles split-generate all versions of the Fukaya category.

$$\mathcal{F}(M_0)^{perf}, \ \mathcal{F}(X_0)^{perf}, \ \mathcal{F}(X_0, B)^{perf}$$

This has to do with Seidel's theory of spherical twists.

Lemma 1 implies A_∞-equivalences

$$\mathcal{F}(M_0)^{perf} \cong \mathcal{F}_{64}^{perf} \qquad \mathcal{F}(X_0)^{perf} = \left(\mathcal{F}_{64,q}^{*}\right)^{perf}$$

$$\mathcal{F}(X_0, B) = \mathcal{F}_{64,q}^{perf}.$$

<u>Lemma 2</u> The deformation class of $\mathcal{F}(X_0,B)$ in $HH^2(\mathcal{F}(M_0), \mathcal{F}(M_0))$ is nontrivial.

The deformation class of $\mathcal{F}_{64,q}$ in $HH^2(\mathcal{F}64, \mathcal{F}_{64})$ is nontrivial.

The first assertion implies the second. The first is proved by showing that a certain object gets nontrivially deformed.

<u>Proposition 3</u> $\mathcal{F}_{64} \cong \mathcal{Q}_{64}$.

To show this, we actually divide by a Γ_{16} action on M_0, and work in the quotient.

Let $\Gamma_4^* = \{\pm I, \pm i I\} \subset SL_4(\mathbb{C})$

Then $M = \mathbb{P}^3 \setminus X_\infty \cong (\mathbb{C}^*)^3 \cong \{x \in \mathbb{C}^4 \mid x_0x_1x_2x_3 = 1\} / \Gamma_4^*$

Now $\Gamma_{16}^* \subset PSL_4(\mathbb{C})$ consisting diagonal mat. of order 4 acts on $\mathbb{P}^3$, M and preserves the kähler structure, and $\pi_4 : M \to \mathbb{C}$

$\overline{M} = M / \Gamma_{16}^*$ and the induced map is

$\overline{M} = \{u_0u_1u_2u_3 = 1\} \qquad \pi_{\overline{M}}(u) = u_0 + u_1 + u_2 + u_3$

$u_i = x_i^k$

This $\pi_{\overline{M}}$ has 4 vanishing cycles. Each one has 16 lifts giving the 64 ones from M.

By thinking about the topology of the covering, we can show

<u>Lemma</u> 4 $\mathcal{F}_{64} \cong \mathcal{F}_4 \rtimes \Gamma_{16}$ where $\mathcal{F}_4$ is the A_∞-algebra of the 4 cycles in $\overline{M}$.

<u>Lemma 5</u> $\mathcal{F}_4 \cong \mathcal{Q}_4$.

Seidel uses now his symplectic Picard-Lefschetz machinery to break this down into a series of calculations that can be fed to a computer.

Lemmas 4+5 imply Proposition 3. Then using our Classification results for A_∞-structures compatible with symmetry completes the proof.