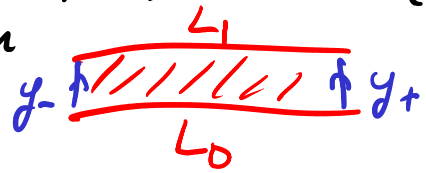
Lecture 17 Proving relations using compactified moduli spaces.

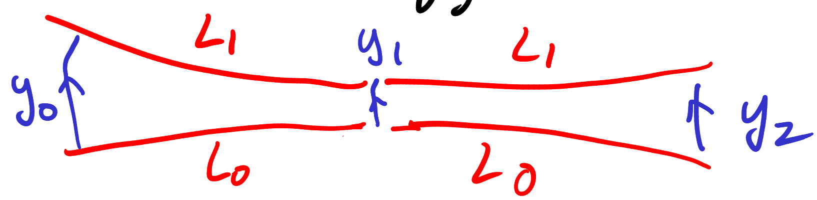
$\partial : CF^{pr}(L_0, L_1) \to CF^{pr}(L_0, L_1)$ counts (mod 2) strips modulo translation



0-dim components

$\mathcal{M}_Z^*(y_-, y_+)^0$

$\partial \circ \partial$ would then count configurations such as

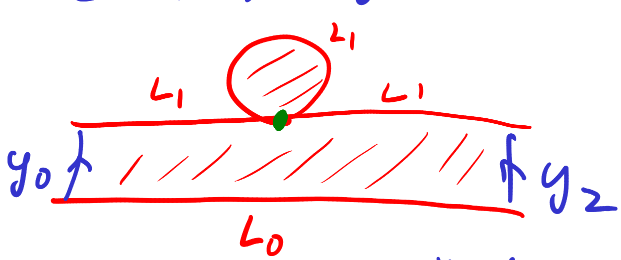


1-dim components.

This configuration is in the boundary of $\overline{\mathcal{M}}_Z^*(y_0, y_2)^1$

If conversely, we knew that every boundary point of $\overline{\mathcal{M}}_Z(y_0, y_2)^1$ corresponds to such a configuration, then the coefficient of y_0 in $\partial(\partial(y_2))$ is zero: This coefficient counts these configurations (mod 2), and a compact 1-manifold with boundary always has an even number of boundary points.

In general, $\overline{\mathcal{M}}_Z^*(y_0, y_2)^1$ may have other boundary points such as



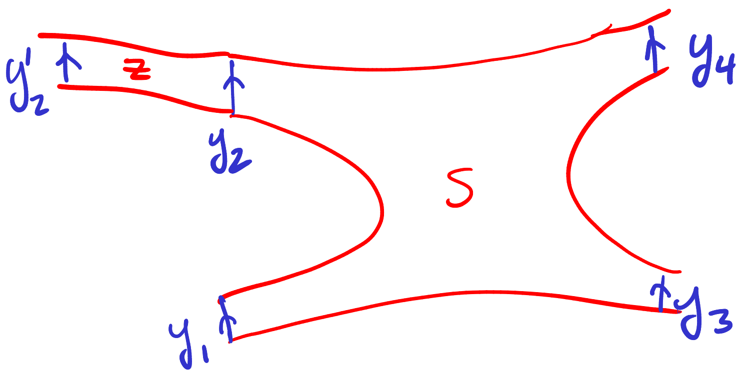
we will later introduce hypotheses that rule out such configurations, making the previous argument valid, and gauranteeing $\partial^2 = 0$.

We denote $HF^{pr}(L_0, L_1) = \ker \partial / \operatorname{Im} \partial$.

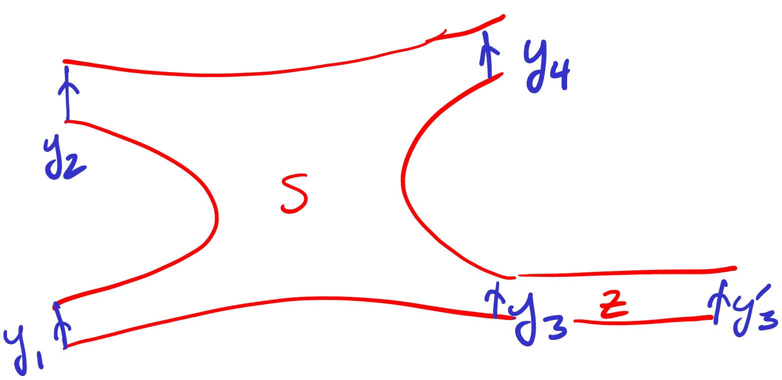
Next, $C\Phi_S : \bigotimes_{\zeta^+} CF^{pr}(L_{\zeta^+,0}, L_{\zeta^+,1}) \longrightarrow \bigotimes_{\zeta^-} CF^{pr}(L_{\zeta^-,0}, L_{\zeta^-,1})$

counts (mod 2) $\mathcal{M}_S(\{y_{\zeta^-}, y_{\zeta^+}\})^0$

The boundary of $\overline{\mathcal{M}}_S(\{y_{\zeta^-}, y_{\zeta^+}\})^1$ contains configurations such as



contributes to $\partial \circ C\Phi_S$



contributes to $C\Phi_S \circ \partial$

Again, once we rule out other degenerations, the fact that $\overline{\mathcal{M}}_S(\{y\})^1$ has an even number of boundary points shows

$$\partial \circ C\Phi_S + C\Phi_S \circ \partial = 0 \pmod 2, \text{ or } \partial \circ C\Phi_S = C\Phi_S \circ \partial$$

so that $C\Phi_S$ is a chain map, and indues a map

$$\Phi_S : \bigotimes_{\zeta^+} HF^{pr}(L_{\zeta^+,0}, L_{\zeta^+,1}) \longrightarrow \bigotimes_{\zeta^-} HF^{pr}(L_{\zeta^-,0}, L_{\zeta^-,1})$$

which are the TFT maps.