MATH 595—HOMOLOGICAL MIRROR SYMMETRY—SPRING 2018

This course is actually two half-semester courses. Students must register for the two halves separately. An enrollment threshold must be met for the second half to run.

Course meets: MWF 1:00–1:50 p.m.

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Course web page: `faculty.math.illinois.edu/~jpascale/courses/2018/595`

Description: Homological Mirror Symmetry (HMS) is the study of the relations between three types of mathematical objects:

$$\text{symplectic manifolds} \longleftrightarrow \text{triangulated categories} \longleftrightarrow \text{algebraic varieties}$$

For a symplectic manifold X, there is a triangulated category $\mathcal{F}(X)$ called the Fukaya category, and for an algebraic variety Y there is a triangulated category $\mathcal{D}(Y)$ called the derived category. We then pose the problem of finding pairs X and Y such that

$$\mathcal{F}(X) \cong \mathcal{D}(Y)$$

The origin of this relation is in theoretical particle physics, where the two categories are interpreted as collections of D-branes, and the relation expresses the duality between A-twisted topological string theory on X and B-twisted topological string theory on Y.

The investigation of this relation raises many questions. How are the two sides actually defined? How do we compute the two sides, and what should the "answer" of such a computation look like? What general structure is present that constrains the problem? The goal of this course is to set up the machinery and understand the solution in a specific case: when X is a hypersurface in projective space, including the quintic threefold, following Seidel and Sheridan. Topics to include:

- Categories: triangulated, differential graded, A_∞.
- Algebraic varieties, categories of coherent sheaves.
- Symplectic manifolds, Lagrangian Floer cohomology, Fukaya categories.
- Case of surfaces, HMS for the two-torus, other relatively simple models.
- Hypersurfaces in projective space.

Prerequisites: This is an advanced topic that connects to many areas of mathematics, so I could list many things here. However, that would defeat the purpose of this course, which is to give students access to this area of research. That said, some prior knowledge of abstract algebra and differential topology is necessary to get anything out of this course. The course on Symplectic Geometry taught concurrently by Prof. Tolman would be helpful, but is not required.

Texts:

- P. Aspinwall, *et al.*, Dirichlet branes and mirror symmetry.
- P. Seidel. Fukaya categories and Picard-Lefschetz theory.
- P. Seidel. Homological mirror symmetry for the quartic surface.
- N. Sheridan. Homological mirror symmetry for Calabi-Yau hypersurfaces in projective space.