(1) Suppose \( n \geq 2 \) and let \( H = (V,E) \) be an \( n \)-uniform hypergraph with \( 4^n - 1 \) edges. Show that there is a coloring of \( V \) by four colors so that no edge is monochromatic.

(2) Prove that if there is a real \( p, 0 \leq p \leq 1 \) such that
\[
{n \choose k} p^{(k)} + \frac{n}{t} (1 - p)^{(t)} < 1,
\]
then the Ramsey number \( R(k,t) \) satisfies \( R(k,t) > n \). Using this show that
\[
R(4,t) \geq \Omega \left( \frac{t^3}{(\ln t)^{3/2}} \right).
\]

(3) Prove that there is a positive constant \( c \) so that every set \( A \) of \( n \) non-zero reals contains a subset \( B \subset A \) of size \( |B| \geq cn \) so that there are no \( b_1, b_2, b_3, b_4 \in B \) satisfying
\[
b_1 + 2b_2 = 2b_3 + 2b_4.
\]

(4) Let \( A = (a_{ij}) \) be an \( n \times n \) matrix with all entries from \( \{0,1\} \). We call \( A \) ninefree if there is no \( 3 \times 3 \) submatrix of all ones. (Note: the rows and columns of a submatrix needn’t be consecutive) Let \( f(n) \) denote the maximum number of ones in a ninefree \( n \times n \) matrix. Give a precise theorem giving a lower bound for \( f(n) \) and then analyze asymptotics, including the constant. Hint: consider a random matrix whose each entry is 1 is chosen independently with probability \( p \). At the end you will need to optimize \( p \).

(5) Prove that there is a constant \( c > 0 \) such that for every even \( n \geq 4 \) the following holds: for every undirected complete graph \( K \) on \( n \) vertices whose edges are colored red and blue, the number of alternating Hamilton cycles in \( K \) (i.e., properly edge-colored cycles of length \( n \)) is at most
\[
n^e \frac{n!}{2^n}.
\]

(6) Let \( R_k(G) \) be the greatest integer \( n \) such that edges of \( K_n \) can be colored by \( k \) colors without monochromatic copy of \( G \). Prove that
\[
R_k(K_{3,3}) \geq \Omega \left( \frac{k^3}{\log^3 k} \right).
\]

(7) Let \( X \) be a collection of pairwise orthogonal unit vectors in \( \mathbb{R}^n \) and suppose the projection of each of these vectors on the the first \( k \) coordinates is of Euclidean norm at least \( \epsilon \). Show that \( |X| \leq k/\epsilon^2 \), and this is tight for all \( \epsilon = k/2^r < 1 \).