Homework 1, Spring 2018, Math 584. Should solve 5 out of 6. All students should type their solutions.

1. Suppose there are $m$ red clubs $R_1, \ldots, R_m$, and $m$ blue clubs in a town of $n$ citizens. Assume that these clubs satisfy the following rules:
   (a) $|R_i \cap B_i|$ is odd for every $i$,
   (b) $|R_i \cap B_j|$ is even for every $i \neq j$.
   Prove that $m \leq n$.

2. Find upper and lower bounds for the size of $t$-distance set in $\mathbb{R}^d$.

3. Using linear algebraic (and not ad hoc) methods prove the following: Let $A$ and $B$ two finite point sets in the 3-dimensional space, such that each pairwise distance between points of $A$ and $B$ is the same. Prove that if $|A| \leq |B|$ then $|A| \leq 2$.

4. In a politically correct town there are $n$ women and $n$ men, and every club has the same number of women as men. All clubs have different sets of members and, amazingly, for any two clubs, exactly as many women as men belong to each. At most how many clubs are in this town?

5. Assume that $G_1, \ldots, G_m$ are complete bipartite graphs such that they cover the edge set of a complete graph $K_n$ such that each edge is covered an odd number of times. Prove that $m \geq (n-1)/2$.

6. Assume that there are $m$ clubs in a town with population $n$. The pairwise intersection of clubs should be even, BUT there is no restriction on the sizes of the clubs. Prove that $m \leq 2^{\lfloor n/2 \rfloor} + 1$. 
