Homework 2, Spring 2013, Math 584. Math students should do 5 out of 7, non math students should solve 4 out of 7 problems. Math students should type their solutions.

1. Prove that for the Lovász $\nu$-function of a graph $G$ the following inequality holds:
   \[ \nu(G) \leq \chi(\bar{G}). \]

2. Give the missing details of the proof of Claim B in M28, in particular look the following issues (they might make sense only if you look the proof carefully):
   (i) Prove that for the product, property (i) is satisfied.
   (ii) Prove that for the product, property (ii) is satisfied.
   (iii) Prove that for the dimension of the product, the product of the dimension is an upper bound.

3. In a politically correct town there are $n$ women and $n$ men, and every club has the same number of women as men. All clubs have different sets of members and, amazingly, for any two clubs, exactly as many women as men belong to each. At most how many clubs are in this town?

4. Find a strongly separating system of $n$, which is really close to the lower bound, proved in the class.

5. Prove the following statement. [Note that simpler than using exterior algebras, ‘elementary’ proof exists]:
   Assume that a collection of sets $\{(A_i, B_i)\}$ satisfies that
   (a) $A_i \cap B_i = \emptyset$ for every $i$,
   (b) $A_i \cap B_j \neq \emptyset$ for every $i \neq j$.
   Then
   \[ \sum_i \left( \frac{|A_i| + |B_i|}{|A_i|} \right)^{-1} \leq 1. \]

6. Prove that in Nagy’s coloring:
   (a) If $t$ is 2 or 3 modulo 4, then there is no blue $K_r$ for $r > t - 2$.
   (b) If $t > 15$ then there is no red $K_r$ for $r > (t - 1)/2$.

7. Deduce Sperner Theorem from the result of Exercise 5.