Homework 1, Spring 2013, Math 584. Math students should do 5 out of 6, non math students should solve 4 out of 6 problems. Math students should type their solutions.

1. Suppose there are $m$ red clubs $R_1, \ldots, R_m$, and $m$ blue clubs in a town of $n$ citizens. Assume that these clubs satisfy the following rules:
   (a) $|R_i \cap B_i|$ is odd for every $i$,
   (b) $|R_i \cap B_j|$ is even for every $i \neq j$.
Prove that $m \leq n$.

2. Find upper and lower bounds for the size of $t$-distance set in $\mathbb{R}^d$.

3. Using linear algebraic (and not ad hoc) methods prove the following:
   Let $A$ and $B$ two finite point sets in the 3-dimensional space, such that each pairwise distance between points of $A$ and $B$ is the same. Prove that if $|A| \leq |B|$ then $|A| \leq 2$.

4. In the class we had the rectangular process (0,1’s are on the vertices of an $n$ by $n$ grid, and a 0 is updated to 1 if it is on the corner of a rectangle whose other vertices are already 1.) Give a CORRECT and COMPLETE proof for the question that how many 1’s are needed to change everthing to 1.

5. Assume that $G_1, \ldots, G_m$ are complete bipartite graphs such that they cover the edge set of a complete graph $K_n$ such that each edge iscovered an odd number of times. Prove that $m \geq (n-1)/2$.

6. Assume that there are $m$ clubs in a town with population $n$. The pairwise intersection of clubs should be even, BUT there is no restriction on the sizes of the clubs. Prove that $m \leq 2^{\lfloor n/2 \rfloor} + 1$. 
