Math 482  (take home) Homework 3

Name: ________________________________

Due Friday, April 6, 2018

Students for three credit hours should solve any five of the following six problems. Students for four credit hours must solve all six problems.

1. We say that here is a tie when selecting the pivot row if, supposing \( x_s \) is the pivot column, there is more than one row \( r \in [m] \) such that \( a_{r,s} > 0 \) and

\[
\frac{a_{r,0}}{a_{r,s}} \leq \frac{a_{i,0}}{a_{i,s}} \quad \text{for all \( i \in [m] \) such that \( a_{i,s} > 0 \)}.
\]

Prove that if there is a tie when selecting the pivot row, then the basic feasible solution corresponding to the next tableau is degenerate.

2. Solve the LP in Example 2.7 (pages 51–52) of the book using Bland’s anticycling algorithm (see Section 2.7). (You do not need to write down the steps that are the same in as the book.)

3. Use the first phase of the two-phase simplex method to show that the following linear program is infeasible.

Minimize 
\[
z = 3x_1 + x_2 + 2x_3
\]
subject to
\[
\begin{align*}
x_1 + 3x_2 + 5x_3 - x_4 &= 10, \\
2x_1 - 2 - 9x_3 - x_4 &= 1, \\
4x_1 + 5x_2 + x_3 + x_4 &= 7, \\
x_1, \ldots, x_4 &\geq 0.
\end{align*}
\]

4. Suppose that you are solving an LP in standard form with 5 variables \( x_1, \ldots, x_5 \) and 2 constraints and the following objective function

\[
\min z = x_1 + 7x_2 + 5x_3 + x_4 + 6x_5.
\]

You add two artificial variables \( y_1 \) and \( y_2 \) and after the first phase of the two-phase simplex method you have the following tableau

\[
\begin{array}{ccccccccc}
& x_1 & x_2 & x_3 & x_4 & x_5 & y_1 & y_2 \\
-x_\xi & 0 & 0 & 8 & 13 & 14 & 4 & 0 \\
x_1 & 10 & 1 & 2 & 2 & 4 & 5 & 1 & 0 \\
y_2 & 0 & 0 & -8 & -13 & -14 & -3 & 1
\end{array}
\]

Drive the artificial variable out of the basis and then solve the original linear program by doing phase two of the simplex method.
5. Introduce 3 artificial variables and solve with two-phase simplex algorithm the LP represented by the tableau below.

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
<th>$x_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-z$</td>
<td>0</td>
<td>4</td>
<td>8</td>
<td>14</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

6. Suppose that there exists $x_0$ and $y$ such that $x_0$ is feasible for the linear program $P$

$$\begin{align*}
\text{min} & \quad c^T x \\
\text{subject to} & \quad Ax = b \\
& \quad x \geq 0
\end{align*}$$

and $y$ satisfies

$$\begin{align*}
c^T y & < 0 \\
Ay & = 0 \\
y & \geq 0.
\end{align*}$$

Prove that $P$ is unbounded. *(Hint: You can prove this directly. You do not need anything from Chapter 3 (Duality) to solve this problem)*