Math 412 **HW3**

Due Friday, February 19, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. For $k \geq 2$, prove that every k-regular bipartite graph has no cut-edge and construct a bipartite graph with all vertex degrees in $\{k, k+1\}$ that has a cut-edge.

2. Given an integer $k \geq 2$, let $G(k)$ be the subgraph of the cube Q_{2k} induced by the vertices in which the number of ones is either k or $k+1$. Compute the number of vertices, the number of edges, and the girth (the length of a shortest cycle) of $G(k)$.

3. Given a nonincreasing list $d = (d_1, \ldots, d_n)$ of nonnegative integers and $1 \leq k \leq n$, let $d(k)$ be obtained from d by deleting d_k and subtracting 1 from the d_k largest elements remaining in the list. Prove that d is graphic if and only if $d(k)$ is graphic. (Hint: Mimic the proof of Havel–Hakimi Theorem.)

4. For every odd n, construct an n-vertex tournament in which every vertex is a king. Does there exist such a tournament with 4 vertices?

5. Let $n \geq 3$ and G be an n-vertex graph. Prove that the following are equivalent.
(A) G is connected and has exactly one cycle.
(B) G is connected and has n edges.
(C) G has exactly one cycle and has n edges.

6. For $n \geq 3$, prove that if an n-vertex graph G has three vertices v_1, v_2, v_3 such that the subgraphs $G - v_1, G - v_2$ and $G - v_3$ are acyclic, then G has at most one cycle.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 1.3: # 1, 4, 5, 9, 12. Section 1.4: # 3, 4, 7. Section 2.1: # 1, 2, 4, 6, 13. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 1.3: # 24, 40, 41, 57, 63. Section 1.4: # 9, 10, 21, 26, 28, 29, 36, 37. Section 2.1: # 15, 19, 27, 29, 31, 44, 52, 53.

Do not write these up!