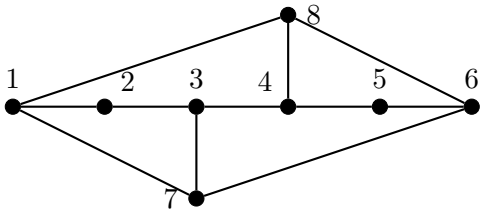
Math 412 **HW2**

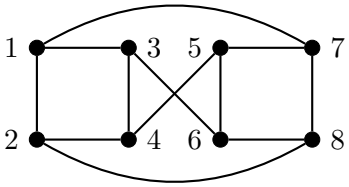
Due Friday, February 5, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. Let P and Q be two maximum length paths in a connected graph G. Prove that P and Q have a common vertex.

2. In each of the graphs below, find a bipartite subgraph with the maximum number of edges. Prove that this is maximum and determine whether this is the only bipartite subgraph with this many edges.





4. Prove that a graph is bipartite if and only if for every subgraph H of G there exists an independent set I in H such that $2|I| \geq |V(H)|$.

3. Prove that if v is a cut-vertex, then $\overline{G} - v$ is connected.

5. (a) Prove that for $n \geq 3$, every connected n-vertex graph contains a maximum independent set that contains all leaves (leaves are vertices of degree 1);
(b) Give an example of a connected graph G with 4 leaves and a maximum independent set I in G that contains only one leaf.

6. Let G be a 7-regular simple graph. (A graph is k-regular if every vertex has degree k). Prove that the set of edges of G cannot be partitioned into paths of length at least 9.

Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section 1.2: # 1, 4, 5, 8, 9, 10, 11. Do not write these up!
OTHER INTERESTING PROBLEMS: Section 1.2: # 14, 18, 23, 41.
Do not write these up!