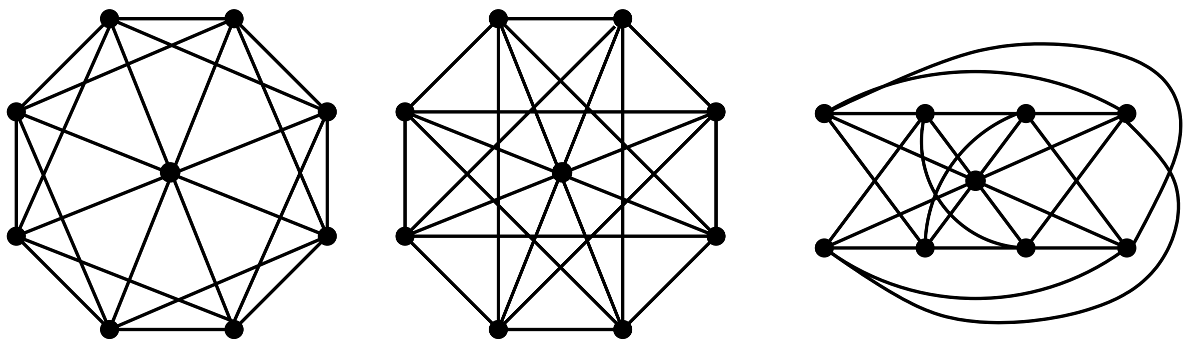
Math 412 **HW1**

Due Friday, January 29, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. Determine which pairs of graphs below are isomorphic.



(a) **(b)** **(c)**

2. Consider the following four families of simple graphs: $A = \{\text{complete graphs}\}$, $B = \{\text{paths}\}$, $C = \{\text{complements of paths}\}$, $D = \{\text{bipartite graphs}\}$. For each pair of these families, determine all graphs that belong to both families in the pair. The word "determine" means, in particular, that you have to prove that there are no other graphs that belong to both families in the pair than your answer gives.

3. Let A be the adjacency matrix of a simple graph G with vertex set $V(G) = \{v_1, \ldots, v_n\}$. Prove that the ith diagonal entry of the matrix $A^2 = A \cdot A$ is the degree of vertex v_i in G. What do the entries in position (i, j) of A^2 say about G?

4. Let $n \geq 4$. For each of the three properties below, describe all n-vertex simple graphs G satisfying this property:
(a) every induced subgraph of G with 3 vertices has either 3 or 2 edges;
(b) every induced subgraph of G with 3 vertices has either 1 or 2 edges;
(c) every induced subgraph of G with 3 vertices has either 0 or 2 edges.

5. Prove that every group of six people contains a set of three people who all know each other or a set of three people who all do not know each other.

6. Prove that if the complete graph on n vertices decomposes into triangles, then either $n - 1$ or $n - 3$ is divisible by 6.

Problems below review basic concepts and their ideas could be used in the tests.

WARMUP PROBLEMS: Section 1.1: # 2, 4, 5, 7, 9, 10. Do not write these up!

OTHER INTERESTING PROBLEMS: Section 1.1: # 11, 18, 20, 22, 23, 24. Do not write these up!