Math 412 **HW6**

Due Wednesday, March 16, 2016

Students in the three credit hour course must solve five of the six problems. Students in the four credit hour course must solve all six problems.

1. A standard card deck has 52 cards with 4 *suits* (hearts, spades, diamonds and clubs) and 13 *values*. The cards are dealt into an array with 4 rows and 13 columns.
(a) Prove that there is a set of 13 cards, one in each column, having distinct values.
(b) Using (a), prove that by a sequence of exchanges of cards with the same value, one can rearrange the cards so that each column contains exactly one heart.

2. In an X, Y-bigraph G, the *deficiency* of a set S is $\text{def}(S) = |S| - |N(S)|$; note that $\text{def}(\emptyset) = 0$. Prove that

$$\alpha'(G) = |X| - \max_{S \subseteq X} \text{def}(S).$$

Hint: Form a bipartite graph G' such that G' has a matching that saturates X if and only if G has a matching of the desired size. Use Hall's Theorem applied to G' to then complete the proof.

3. Use the previous problem to prove the König-Egeváry Theorem.

4. Let G be a bipartite graph. Prove that $\alpha(G) = n(G)/2$ if and only if G has a perfect matching.

5. Let G be a 3-regular graph with at most two cut-edges. Prove that G has a 1-factor.

6. For every graph G, prove that $\beta(G) \leq 2\alpha'(G)$. For each $k \in \mathbb{N}$, construct a simple graph G with $\alpha'(G) = k$ and $\beta(G) = 2k$.

Problems below review basic concepts and their ideas could be used in the tests.
WARMUP PROBLEMS: Section 3.3: # 1, 2, 4, 6, 7. Do not write these up!

OTHER INTERESTING PROBLEMS:
Section 3.3: # 8, 9, 15, 22, 24, 26. Do not write these up!