For exercises 1, 3 and 4 **no collaborative work is allowed!** Please write each solution to a separate (new) page.

1. Karmarkar:
   (i) Prove or disprove that in Karmarkar’s method always $x^1 = y^1$.
   (ii) Prove or disprove that in Karmarkar’s method always $x^2 = y^2$.
   (iii) Perform two iterations of Karmarkar’s method for the following LP:
   \[
   \min z = 2x^2 \text{ subject to } x_1 + x_2 - 2x_3 = 0, \quad x_1 + x_2 + x_3 = 1, \quad x_1, x_2, x_3 \geq 0.
   \]
   (iv) Prove or disprove that if the input is rational then we maintain rationality.

2. a.) Let $n$ be large and let $E(K_n)$ be equipartitioned into 2 parts $V_1, V_2$ and define a coloring $c_1$ so that edges inside of a part are red and edges between the parts are blue. Construct a feasible solution for the LP of $r(c_1; n, k)$ in which all edges in $V_1$ get the same weight, all those in $V_2$ get the same weight and all those in $(V_1, V_2)$ get the same weight. For 2a. and 2b., construct the best feasible solutions with the given restrictions.
   b.) Let $n$ be large and let $E(K_n)$ be equipartitioned into 5 parts $V_1, V_2, V_3, V_4, V_5$ and define a coloring $c_2$ so that edges inside of a part are colored arbitrarily but $(V_i, V_{i+1})$ (indices are mod 5) are red and $(V_i, V_{i+2})$ are blue. Construct a feasible solution for the LP of $r(c_2; n, k)$ in which all edges in $V_i$ get weight zero and all edges in the same $(V_i, V_j)$ get the same weight. For 2a. and 2b., construct the best feasible solutions with the given restrictions.
   c.) Compare the value of the solution in 2a) to that in 2b). Note, your answer will depend on $k$.
   d.) Prove that your weight assignment in part 2.a) is asymptotically optimal over all weight assignments, for $n$ sufficiently large and $k = 3$. In other words, prove that the value of the LP you find in 2a., for $k = 3$, is at least $r(c_1; n, 3) - o(n^2)$. Hint: Treat R and B independently.
   e.) [It could be hard] Do 2d.) for arbitrary $k > 3$.

3. Suggest a homework problem which was not assigned, but should have. Provide a solution.

4. Suggest a (different!) homework problem which was not assigned, but should have. Provide a solution.