MATH 580, FALL 2012 - HOMEWORK 11

WARMUP PROBLEMS: Section 10.1 #1, 3, 6. Section 10.2 #3. Section 10.3 #1. Section 11.1 #4, 5. Do not write up!

EXTRA PROBLEMS: Section 10.1 #21, 30, 33. Section 10.2 #5, 7, 10, 12, 13, 18, 19, 31, 33. Section 10.3 #4, 7, 8. Section 11.1 #7, 8, 9, 13, 17, 19. Think about some.

WRITTEN PROBLEMS: Do five of the following six. Due Wednesday, November 28.

1. Let S be a set of $R(m, m; 3)$ points in the plane no three of which are collinear. Prove that S contains m points that form a convex m-gon.

2. *Ramsey graph theory.*

a) Determine the Ramsey number $R(K_{1,r}, K_{1,s})$ (in all cases).

b) Let T be a tree with m edges, and let n be a multiple of m. Determine the Ramsey number $R(T, K_{1,n+1})$.

3. Show that for large n, every k-coloring of the nonempty subsets of $[n]$ gives the same color to some two disjoint sets and their union. (Hint: Take $n \geq R_k(3; 2)$, as defined in Definition 10.2.10, and consider the proof of Theorem 10.3.1. One need not use all the nonempty subsets of $[n]$.)

4. Consider the equation $x_1 + \cdots + x_m = x_{m+1}$, for $m \geq 2$.

a) Determine the smallest integer n such that every 2-coloring of $[n]$ contains a monochromatic solution (numbers may be repeated on the left in forming a monochromatic solution).

b) For each $n \in \mathbb{N}$, determine the maximum size of a subset of $[n]$ not containing a solution to the above equation.

5. *Turán's proof of Turán's Theorem.* Recall that $t_r(n) = |E(T_{n,r})|$.

a) Prove that a maximal graph with no $(r+1)$-clique has an r-clique.

b) Prove that $t_r(n) = \binom{r}{2} + (n-r)(r-1) + m_{n-r,r}$.

c) Use parts (a) and (b) to prove Turán's Theorem by induction on n, including the uniqueness of graphs achieving the bound.

6. Let G be a graph on n vertices such that $\overline{G}$ has no triangles. Prove that G has at least $\binom{\lfloor n/2 \rfloor}{3} + \binom{\lfloor (n+1)/2 \rfloor}{3}$ triangles and that this is sharp for $n > 9$.