MATH 580, FALL 2012 - HOMEWORK 10

WARMUP PROBLEMS: Section 9.1 #2–9. Section 9.2 #1, 3, 4, 5. Section 9.3 #3, 4, 5.

EXTRA PROBLEMS: Section 9.1 #6, 10, 11, 19, 28, 29, 34, 37. Section 9.2 #6, 8, 9, 11. Section 9.3 #7, 10, 11, 18, 20, 36, 38. Do not write up.

WRITTEN PROBLEMS: Do five of the following six. Due Friday, November 16.

1. Use Euler's Formula to count the regions formed by n lines in the plane, assuming that no two are parallel and no three have a common point.

2. *Structure of Eulerian plane graphs.* Let G be a connected plane graph whose vertex degrees are all even. Prove the following statements.

a) G has an Eulerian circuit that does not cross itself.

b) If also every bounded face is a triangle, then $|E(G)|$ is divisible by 3.

c) If also G is a maximal outerplanar graph, then $|V(G)|$ is divisible by 3.

3. Prove that every 3-connected graph with at least six vertices that contains a subdivision of K_5 also contains a subdivision of $K_{3,3}$.

4. *Short proof of the Five Color Theorem.*

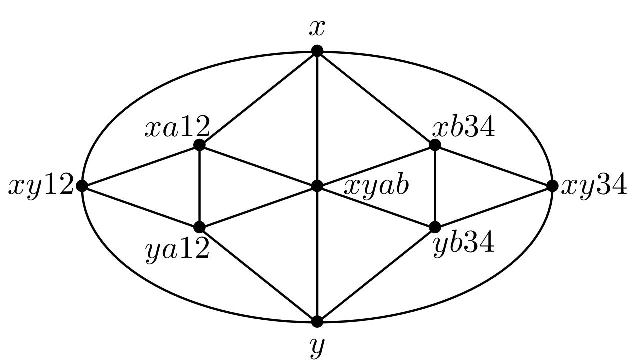
a) Let v be a 5-vertex in a plane graph G. Let x and y be nonadjacent neighbors of v, and let G' be the graph obtained from G by contracting the edges vx and vy. Prove that if G' is 5-colorable, then G is 5-colorable.

b) Use part (a) to give a short inductive proof of the Five Color Theorem.

5. *Non-4-choosable planar graph of order 75.*

a) Prove that the graph below cannot be properly colored from the given lists, where the 5-valent vertices have lists of size 1 and the others have lists of size 4.

b) Use part (a) to construct a 3-colorable planar graph on 75 vertices that is not 4-choosable. (Hint: It may help first to construct such a graph with 114 vertices and then one with 86 vertices.)



6. Let G be a connected plane graph such that $\delta(G), \delta(G^*) \geq 3$. Use balanced discharging to prove that G has a vertex of degree 3 on a face of length at most 5 or a vertex of degree at most 5 on a triangle. (Comment: The Platonic solids show that all five resulting configurations are needed.)