MA TH 580, FALL 2012 - HOMEWORK 10

WARMUP PROBLEMS: Section 9.1 #2–9. Section 9.2 #1, 3, 4, 5. Section 9.3 #3, 4, 5.
EXTRA PROBLEMS: Section 9.1 #6, 10, 11, 19, 28, 29, 34, 37. Section 9.2 #6, 8, 9, 11.
Section 9.3 #7, 10, 11, 18, 20, 36, 38. Do not write up.
WRITTEN PROBLEMS: Do five of the following six. Due Friday, November 16.

1. Use Euler’s Formula to count the regions formed by \( n \) lines in the plane, assuming that no two are parallel and no three have a common point.

2. Structure of Eulerian plane graphs. Let \( G \) be a connected plane graph whose vertex degrees are all even. Prove the following statements.
   a) \( G \) has an Eulerian circuit that does not cross itself.
   b) If also every bounded face is a triangle, then \(|E(G)|\) is divisible by 3.
   c) If also \( G \) is a maximal outerplanar graph, then \(|V(G)|\) is divisible by 3.

3. Prove that every 3-connected graph with at least six vertices that contains a subdivision of \( K_5 \) also contains a subdivision of \( K_{3,3} \).

4. Short proof of the Five Color Theorem.
   a) Let \( v \) be a 5-vertex in a plane graph \( G \). Let \( x \) and \( y \) be nonadjacent neighbors of \( v \), and let \( G' \) be the graph obtained from \( G \) by contracting the edges \( vx \) and \( vy \). Prove that if \( G' \) is 5-colorable, then \( G \) is 5-colorable.
   b) Use part (a) to give a short inductive proof of the Five Color Theorem.

5. Non-4-choosable planar graph of order 75.
   a) Prove that the graph below cannot be properly colored from the given lists, where the 5-valent vertices have lists of size 1 and the others have lists of size 4.
   b) Use part (a) to construct a 3-colorable planar graph on 75 vertices that is not 4-choosable. (Hint: It may help first to construct such a graph with 114 vertices and then one with 86 vertices.)

6. Let \( G \) be a connected plane graph such that \( \delta(G), \delta(G^*) \geq 3 \). Use balanced discharging to prove that \( G \) has a vertex of degree 3 on a face of length at most 5 or a vertex of degree at most 5 on a triangle. (Comment: The Platonic solids show that all five resulting configurations are needed.)