HW11, DUE NOV 11

**Exercise 1)** Let $I_1, I_2$ be intervals. Let $f: I_1 \to I_2$ be a bijective function and $g: I_2 \to I_1$ be the inverse. Suppose that both $f$ is differentiable at $c \in I_1$ and $f'(c) \neq 0$ and $g$ is differentiable at $f(c)$. Use the chain rule to find a formula for $g'(f(c))$ (in terms of $f'(c)$).

**Exercise 2)** Suppose that $f: \mathbb{R} \to \mathbb{R}$ is a differentiable function such that $f'$ is a bounded function. Then show that $f$ is a Lipschitz continuous function.

**Exercise 3)** Suppose that $f: \mathbb{R} \to \mathbb{R}$ is a function such that $|f(x) - f(y)| \leq |x - y|^2$ for all $x$ and $y$. Show that $f(x) = C$ for some constant $C$. Hint: show that $f$ is differentiable at all points and compute the derivative.

**Exercise 4)** Suppose that $I$ is an interval and $f: I \to \mathbb{R}$ is a differentiable function. If $f'(x) > 0$ for all $x \in I$, show that $f$ is strictly increasing.

**Exercise 5)** Suppose that $I$ is an interval and $f: I \to \mathbb{R}$ is a differentiable function. Show that $f'(x) \leq 0$ for all $x \in I$ if and only if $f$ is decreasing.

**Exercise 6)** Suppose $f: (a, b) \to \mathbb{R}$ is a differentiable function such that $f'(x) \neq 0$ for all $x \in (a, b)$. Suppose that there exists a point $c \in (a, b)$ such that $f''(c) > 0$. Prove that $f'(x) > 0$ for all $x \in (a, b)$.

**Exercise 7)** Suppose that $f: [-1, 1] \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} 1 & \text{if } x > 0, \\ 0 & \text{if } x \leq 0. \end{cases}$$

Prove that $f \in \mathcal{R}[-1, 1]$ and compute $\int_{-1}^{1} f(x) \, dx$ using the definition of the integral.