Exercise 1) Show that $f : (c, \infty) \to \mathbb{R}$ for some $c > 0$ and defined by $f(x) = \frac{1}{x}$ is Lipschitz continuous.

Exercise 2) Show that $f : (0, \infty) \to \mathbb{R}$ defined by $f(x) = \frac{1}{x}$ is not Lipschitz continuous.

Exercise 3) Let $f : (0, 1) \to \mathbb{R}$ be a bounded continuous function. Show that the function $g(x) = x(1 - x)f(x)$ is uniformly continuous.

Exercise 4) Let $f : Q \to \mathbb{R}$ be a uniformly continuous function. Show that there exists a uniformly continuous function $\tilde{f} : \mathbb{R} \to \mathbb{R}$ such that $f(x) = \tilde{f}(x)$ for all $x \in Q$.

Exercise 5) Prove the product rule for derivatives. Let $I$ be an interval, let $f : I \to \mathbb{R}$ and $g : I \to \mathbb{R}$ be differentiable functions. If $h : I \to \mathbb{R}$ is defined by

$$h(x) := f(x)g(x),$$

then $h$ is differentiable and

$$h'(x) = f(x)g'(x) + f'(x)g(x).$$

Hint: Write $f(x)g(x) - f(c)g(c) = f(x)(g(x) - g(c)) + g(c)(f(x) - f(c))$.

Exercise 6) Prove that a polynomial is differentiable and find the derivative. Hint: Use the product rule and the linearity of the derivative and induction.

Exercise 7) Let $f : I \to \mathbb{R}$ be differentiable. Define $f^n$ be the function defined by $f^n(x) = (f(x))^n$. Prove that $(f^n)'(x) = n(f(x))^{n-1}f'(x)$.