These problems may (or may not) be longer / harder than real test problems. These are just extra problems, use practice exams 1-3 and the real exams 1-3 for practice.

1. Let \( x = \cos t, \ y = \sin t, \) and \( z = t^3 \) define a curve.
   (a) Find the tangent line to the curve at the point corresponding to \( t = \frac{\pi}{2} \).
   (b) Find the equation for the plane normal to the curve at the point corresponding to \( t = \frac{\pi}{2} \).

2. Suppose a cylinder of radius 1 meter and length 10 meters has density \((1 + d^2) \frac{kg}{m^3}\), where \( d \) is the distance from one end of the cylinder.
   (a) Set up (but do not yet compute) a triple iterated integral for the mass of the cylinder.
   (b) Compute the mass of the cylinder in kg.
   \(\text{Hint: use cylindrical coordinates}\)

3. Let \( f(x) = \int_{\sin(x)}^{\cos(x)} e^t x \ dt \)
   Compute \( f'(x) \): 

4. Let \( C \) be the curve given by \( z = \sin(2\pi x), \ y = \cos(2\pi x) \), for \( 0 \leq x \leq 1 \), oriented in the direction of positive \( x \).
   Compute: \( \int_C 2xyez \ dx + x^2e^z \ dy + x^2y^2e^z \ dz \)

5. Let \( C \) be the closed curve given by \( x = \cos(t), \ y = \sin(t), \ z = \cos^2(t) \), \( 0 \leq t \leq 2\pi \) oriented in the direction of increasing \( t \). Let \( F(x, y, z) = e^{x^2+y^2+z^2} \)
   Compute: \( \int_C \nabla F \ ds \)

6. Let \( S \) be the surface \( x^2 + y^2 = 1 \) and \( 0 \leq z \leq 1 \) (just the side of the cylinder not the caps on the top and bottom). Let \( \mathbf{v} = xyz \mathbf{i} + e^{x^2+y^2+z^2} \mathbf{j} + (z+1)(x^2+y^2) \mathbf{k} \) and let \( \mathbf{n} \) be the outer normal.
   Compute \( \iint_S \text{curl} \mathbf{v} \cdot \mathbf{n} \ d\sigma \)

7. Find the equation for the plane that best approximates \( z = x^3 + 3xy + y^3 \) at the point where \( x = 1 \) and \( y = 2 \).

8. Let \( \psi = \log \frac{1}{\sqrt{x^2+y^2}} \) be the potential function in a plane and let \( \mathbf{F} = \nabla \psi \) be the corresponding force field. Let \( C \) be the unit circle \( x^2 + y^2 = 1 \). Let \( \mathbf{n} \) be the unit outer normal.
   (a) Compute \( \oint_C F_n \ ds = \int_C \mathbf{F} \cdot \mathbf{n} \ ds \)
   (b) Compute \( \oint_C F_T \ ds \)